STUDIES ON A NEW COSMOLOGICAL MODEL BASED ON COMPLEX METRIC

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Abstract

In this thesis, the implications of a new cosmological model are studied, which has features similar to that of decaying vacuum cosmologies. Decaying vacuum (or cosmological constant Λ) models are the results of attempts to resolve the problems that plague the standard hot big bang model in cosmology - the problems which elude a satisfactory solution even after the two decades of the advent of inflationary models, the first and much publicised cure to them. We arrive at the present model by a radically new route, which extends the idea of a possible signature change in the metric, a widely discussed speculation in the current literature. An alternative approach uses some dimensional considerations in line with quantum cosmology and gives an almost identical model. Both derivations involve some fundamental issues in general theory of relativity. The model has a coasting evolution (i.e., $a \propto t$). It claims the absence of all the aforementioned puzzles in the standard model and has very good predictions for several measurable quantities. In the first two chapters of the thesis, we review the general theory of relativity, the standard model in cosmology, its successes, the problems in it and also the most successful of those attempts to solve them, namely, the inflationary and decaying vacuum models. In the third chapter, we present and discuss the new cosmological model in detail. The fourth chapter is concerned with quantum cosmology. We briefly review the canonical quantisation programme of solving the Wheeler-DeWitt equation, apply the procedure to our model and show that it satisfies many of the much sought-after ideals of this formalism. The last chapter of the thesis discusses the solution of Einstein equations in the new model in comparison with other ones, its connection with other coasting models, the appearance of a Casimir type negative energy density in it and also the prospects and challenges ahead for the model.

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I dedicate this work to my father and mother.

Preface

Einstein's general theory of relativity (GTR) is perhaps the profoundest theory concerning the physical world with regard to its revolutionary content and the highly sophisticated mathematical apparatus necessitated by it. While most theories of nature evolved as part of experimental and observational encounters with physical situations by innumerable scientists through generations, this theory, in its complete form was conceived almost single handedly by this intellectual giant and was much ahead of its time. Prospects of putting it to direct test may ever remain poor, but the theory assumes a central role in interpreting astrophysical and cosmological data. In fact, the perspective of mankind on the cosmos was carried to unforeseen heights in so short a period in this century mainly due to GTR.

On the other hand, cosmology has never enjoyed the same status as physics or astronomy till recent times, partly due to its speculative nature and partly due to the lack of adequate observational data. But during the past decade, with the launching of 'Hubble Space Telescope' and the 'Cosmic Background Explorer' (COBE) satellite, a wealth of information is pouring in from the deep skies. But since we cannot experiment with the cosmos, one can only resort to model-making and then to check how far the observational data agree with the predictions of the model. The most successful cosmological model, with the least amount of speculatory inputs and maximum consistency with observational facts is considered to be the 'standard' or the 'hot big bang' model. The model predicts an early hot phase for the universe, the relic of which is the cosmic background radiation. In addition to background radiation, it predicts the Hubble expansion and also the observed abundance of the light nuclei in the universe.

However, there are certain problems in this picture, which are identified and given serious attention in the past few years. Some of these are directly dependent upon the simplifying assumptions taken and some of them arise while trying to incorporate the ideas of particle physics theories into the standard model. But there are problems like the singularity, horizon, flatness and cosmological constant problems which exhibit genuine inconsistencies in the model and require substantial modifications in it. One of the most widely discussed such modifications to standard model is the 'inflation', which brings in the possibility of an exponential expansion of the universe in its early evolution, caused by the potential energy of a scalar field. This scenario can successfully handle many of the problems, but does not solve the singularity and cosmological constant problems and also brings in a new 'age' problem. Recently, some alternative cosmological models have gained considerable attention in the literature under the title 'decaying-λ cosmologies'. They have a time-varying cosmological constant, which helps to solve also the cosmological constant problem, in addition to those ones inflation can solve.

In this thesis, we study the implications of a new cosmological model, which has features similar to those of decaying-λ cosmologies. Apart from the presence of a time-varying cosmological constant, the model has an evolution and thermal history quite close to that of the standard model. At the same time, it claims the absence of all out-

standing problems in that model and has very good predictions for several measurable quantities. We arrive at this model by extending the idea of a possible signature change' in the early universe, a widely discussed speculation which involves some basic issues in the GTR. This extension leaves us in an unphysical universe, but we have noticed that a proper interpretation of the theory will enable us to obtain an excellent cosmological model, with the essential features as summarised above. For the purpose of comparison, we begin the thesis by introducing the developments in the field of cosmology, starting from the fundamentals. In the first two preliminary chapters, we review the GTR, the standard model in cosmology, its successes, the problems in it and also the most successful of those attempts to solve these problems, namely, inflation and decaying vacuum models. Following this, we present the new cosmological model in Ch. 3 and discuss its important features like thermal evolution, avoidance of cosmological problems, prediction of observable quantities, etc..

The fourth chapter is devoted to quantum cosmology. Quantum cosmology is the result of attempts to reconcile GTR and quantum mechanics, the other major breakthrough in physics during this century. This subject, which is still in its infancy, has a more direct bearing on the conceptual foundations of physics. One route to this goal is to write the wave equation for the universe (Wheeler-DeWitt equation). We briefly review the achievements in this direction and then apply the procedure to the new cosmological model. It is shown that the programme works exceedingly well in the new context.

The concluding chapter of the thesis presents a discussion of the new model in comparison with the other cosmological models.

Except in one subsection, we use natural units (in which $\hbar = c = k_B = 1$) throughout in the derivations. But when explicit calculations are made, we convert the final results into conventional units with the help of a table. Notations, sign conventions etc. are adopted mostly the same as that in [1]. Specifically, we use Latin indices $i, j, .. = 0, 1, 2, 3$ and Greek indices $\mu, \nu .. = 1, 2, 3$.

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.... These games do not compliment or contradict cognitive reason. Therefore it is clear how they cannot be worked. It cannot be an ontological reconciliation in which there are several aspects of being nor an epistemological one which assumes several types of knowledge. Working this terrain pursues a fore-sight, a horizon of expectations; not definitely of one goal to truth, not even of many roads to one truth, but perhaps, of many roads to many truths, some to nowhere.

Muralidharan M. in

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Chapter 1

Relativistic Cosmology

Öpic in 1922 measured the distance to the Andromeda nebula to be nearly equal to 450 Kpc, which when compared to the measured radius ≈ 8 Kpc of our own Milky-way is enormous. This was conclusive proof of the fact that those observed spiral nebulae like that of Andromeda are in fact island universes (galaxies), with a size comparable to that of the Milky-way galaxy. Also it was the first believable evidence that the universe extends to scales well above that of our galaxy. The emergence of modern observational cosmology, with the notion of galaxies as basic entities distributed over space, can be traced back to this event. Around the same time, Slipher has measured the spectral displacement of forty-one nearby galaxies and thirty-six amongst them showed redshift. In 1929 Hubble, on the basis of Slipher's observations, proposed a linear relation - Hubble law - between the distances to galaxies and their redshifts. The next landmark in observational cosmology was the discovery of the cosmic microwave background radiation (CMBR) by Penzias and Wilson in 1965. Detailed observations on these three phenomena [2], namely, distribution of galaxies, variation of galaxy redshifts with distance and CMBR still remain the pillars of observational cosmology.

Clearly, these observations require interpretations for any progress to be made. The best thing one can do is to make a model by extrapolating tested theories to the realm of cosmology and compare the predictions of the model with more detailed observations. However, this procedure involves certain judicious choices and assumptions. At the range of scales involved, gravity is the only known interaction to be counted and the most refined and tested theory of gravity is Einstein's general theory of relativity (GTR) [1]-[6]. We discuss only models which use GTR or some slight variants of it and hence a very brief review of this theory is presented in Sec. 1.1. Again, the application of GTR to cosmology requires some simplifying assumptions for any predictions to be made. First of all, we assume the cosmological principle to be valid; i.e., at any given cosmic time, the distribution of galaxies in the universe is assumed to be homogeneous and isotropic at sufficiently large scales and also that the mean rest frame of galaxies agrees with this definition of simultaneity. In Sec. 1.2, we review models of the universe obeying the cosmological principle, with different models having different matter content. The last

section in this chapter is devoted to a brief review of the most popular, standard hot big bang model. We explain how the model accounts for the observed facts at large, for the benefit of comparison with the new cosmological model to be presented in this thesis.

1.1 General Theory of Relativity

The conventional route to GTR is to start from the observed phenomenon of the equality of gravitational and inertial masses of objects and then to elevate this equality to the 'principle of equivalence'. But this theory, which is primarily a geometric theory - in the sense that gravitational field can be represented by the metric tensor and freely falling bodies move along geodesics - can be deduced also from an action principle. For our purpose of introducing a new cosmological model based on a complex metric, it is convenient to adopt the latter approach. We first derive, by varying an action, the equations of motion and the field equations in GTR, making explicit the form of the energy-momentum tensor for various types of matter. Then we make use of the opportunity to introduce Einstein's famous cosmological constant, as it plays an important part in our subsequent discussions. Lastly, by using the 3+1 split of spacetime, it is described how to identify a suitable Lagrangian density in this case, so as to enable writing the field equations as Euler-Lagrange equations.

1.1.1 Field Equations

GTR is a theory of gravity which follows by requiring that the action [1], [3]-[6]

$$I = \frac{-1}{16\pi G} \int R(g_{ik}) \sqrt{-g}\, d^4x + \int \Lambda \sqrt{-g}\, d^4x \equiv I_G + I_M \tag{1.1}$$

be stationary under variation of the dynamical variables in it. I is called the Einstein-Hilbert action. The first integral is the gravitational action I_G where $R(g_{ik})$ is the curvature scalar, g_{ik} are the covariant components of the metric tensor of the 4-dimensional spacetime, defined by the expression for the line element

$$ds^2 = g_{ik} dx^i dx^k \tag{1.2}$$

and $g \equiv \det(g_{ik})$. $R(g_{ik})$ is given by

$$R = g^{ik} R_{ik}, \tag{1.3}$$

where the g^{ik} are the contravariant components of the metric tensor and R_{ik} is the Ricci tensor

$$R_{ik} = g^{lm} R_{limk} = R^l_{ilk}. \tag{1.4}$$

In the above, R^l_{ilk} is the contracted form of the Riemann tensor

$$R^l_{imk} = \frac{\partial \Gamma^l_{ik}}{\partial x^m} - \frac{\partial \Gamma^l_{im}}{\partial x^k} + \Gamma^l_{nm}\Gamma^n_{ik} - \Gamma^l_{nk}\Gamma^n_{im} \tag{1.5}$$

and lastly, the Christoffel symbols $\Gamma^{\,l}_{ik}$, in terms of the metric tensor are defined as

$$\Gamma^l_{ik} = \frac{1}{2} g^{lm} \left(\frac{\partial g_{mi}}{\partial x^k} + \frac{\partial g_{mk}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^m} \right). \tag{1.6}$$

In the second integral in Eq. (1.1), which is the matter action I_M, Λ corresponds to the matter fields present. A general expression for Λ is of the form

$$\Lambda = \Lambda(\phi^A, \phi^A_{,i}, x^i), \tag{1.7}$$

where ϕ^A $(A = 1, 2, 3..)$ are a series of functions of spacetime coordinates x^i and "$,i$" refers to differentiation with respect to x^i. For example, the electromagnetic field should have

$$\Lambda_{em} = -\frac{1}{16\pi} F_{ik}F^{ik}; \qquad F_{ik} = A_{k,i} - A_{i,k}. \tag{1.8}$$

Here, A_i are the scalar and vector potentials. For the scalar field ϕ which appears in particle physics theories,

$$\Lambda_\phi = \frac{1}{2} g^{ik} \frac{\partial \phi}{\partial x^i}\frac{\partial \phi}{\partial x^k} - V(\phi), \tag{1.9}$$

where $V(\phi)$ is the potential of the field. But for matter in the form of particles, I_M is written in a form different from that in Eq. (1.1). As an example, consider particles interacting with an electromagnetic field. We write the matter action for the system as

$$I_{M,particles} = -\sum_a m_a \int ds_a - \sum_a e_a \int \; A_i \, dx^i + \int \; \Lambda_{em} \, \sqrt{-g} \, d^4x, \tag{1.10}$$

where m_a is the mass, e_a the charge and the summation is over all the particles a. If the action I in (1.1) is minimized by varying only the position of the worldline of a typical particle, keeping its endpoints fixed, we get the equation of motion of the particle in the combined gravitational and other fields with which it interacts. In the above example, the required equations of motion for the particle are

$$\frac{d^2x^i}{ds_a^2} + \Gamma^i_{kl}\frac{dx^k}{ds_a}\frac{dx^l}{ds_a} = \frac{e_a}{m_a} F_l{}^i \frac{dx^l}{ds_a}. \tag{1.11}$$

On the other hand, the equations of motion for the fields, i.e., the field equations are obtained when we minimize the action I by varying only the fields. For example, if we minimize the action with I_M given by equation (1.10) by varying A_i, the Maxwell equations for the electromagnetic field are obtained:

$$F^{ik}_{;i} = 4\pi j^k. \tag{1.12}$$

Here "$;i$" refers to covariant differentiation with respect to x^i. In fact, the form (1.8) for Λ_{em} was chosen in such a way that we obtain this result.

Lastly, the Einstein field equations, i.e., the equations of motion for the gravitational field can be obtained by minimising the action I by varying the metric tensor g_{ik}. Note that this is the only variation which will affect I_G. It can be seen that under the variation $g_{ik} \to g_{ik} + \delta g_{ik}$,

$$\delta I_G \equiv \frac{1}{16\pi G} \int (R^{ik} - \frac{1}{2} R\, g^{ik})\, \delta g_{ik}\, \sqrt{-g}\, d^4x. \tag{1.13}$$

The variation in the the matter action I_M can be written as

$$\delta I_M \equiv -\frac{1}{2} \int T^{ik}\, \delta g_{ik}\, \sqrt{-g}\, d^4x. \tag{1.14}$$

When Λ in (1.1) is of the general form (1.7), T^{ik}, the energy-momentum tensor can be seen to be of the form

$$T^{ik} = 2\left[\frac{1}{\sqrt{-g}}\left(\frac{\partial \Lambda \sqrt{-g}}{\partial g_{ik,l}}\right)_{,l} - \frac{\partial \Lambda}{\partial g_{ik}} - \frac{1}{2}\Lambda g^{ik}\right]. \tag{1.15}$$

For $\Lambda = \Lambda_{em}$ as in (1.8), this gives

$$T^{ik}_{em} = \frac{1}{4\pi}(\frac{1}{4} F_{mn}F^{mn} g^{ik} - F^i_l F^{lk}). \tag{1.16}$$

For $\Lambda = \Lambda_\phi$ as in (1.9), (1.15) gives

$$T^{ik}_\phi = g^{il} g^{km} \frac{\partial \phi}{\partial x^l} \frac{\partial \phi}{\partial x^m} - g^{ik}\Lambda_\phi. \tag{1.17}$$

For matter in the form of particles, as in the case of (1.10), with the four-momentum p^i_a and the energy of the particle E_a,

$$T^{ik}_{particles} = \sum_a \frac{1}{E_a} p^i_a p^k_a \delta^3(x - x_a). \tag{1.18}$$

For a perfect fluid, i.e., fluid having at each point a velocity vector $\mathbf{v}$ such that an observer moving with this velocity sees the fluid around him as isotropic, the above energy-momentum tensor can be cast in the form

$$T^{ik}_{perfect\ fluid} = (p + \rho) U^i U^k - p g^{ik} \tag{1.19}$$

where $U^i \equiv dx^i/ds$.

Combining (1.13) with (1.14) and putting $\delta I = \delta I_G + \delta I_M = 0$, we get the Einstein field equations as

$$G^{ik} \equiv R^{ik} - \frac{1}{2} g^{ik} R = 8\pi G T^{ik}. \tag{1.20}$$

The Einstein equations also imply the energy conservation law

$$T^{i}_{k;l} = 0. \tag{1.21}$$

1.1.2 Cosmological Constant

It is now instructive to see how I_G is chosen in the form as in (1.1) [1]. As usual in writing variational principles, the action shall be expressed in terms of a scalar integral $\int \mathcal{G}\sqrt{-g}\, d^4x$, taken over all space and over the time coordinate $x^0 = t$ between two given values. Since the attempt is to describe the gravitational field in terms of g_{ik}, which are thus the 'potentials', we shall require that the resulting equations of the gravitational fields must contain derivatives of g_{ik} no higher than the second order. For this, $\mathcal{G}$ should contain only g_{ik} and its first derivatives. But it is not possible to construct an invariant $\mathcal{G}$ (under coordinate transformations) using g_{ik} and the Christoffel symbols Γ^i_{kl} (which contain only first derivatives of g_{ik}) alone, since both g_{ik} and Γ^i_{kl} can be made equal to zero at a given point by appropriate coordinate transformations. Thus we choose R in place of $\mathcal{G}$, though R contains second derivatives of g_{ik}. This is sufficient since the second derivatives in R are linear and the integral $\int R\, \sqrt{-g}\, d^4x$ can be written as the sum of two terms: (1) an expression not containing the second derivatives of g_{ik} and (2) the integral of an expression in the form of a four-divergence of a certain quantity. By using Gauss's theorem, the latter can be transformed into an integral over a hypersurface surrounding the four-volume over which the integrations are performed. When we vary the action, the variations of the second term vanish since by the principle of least action, the variation of the field g_{ik} at the limits of the region of integration are zero. Thus $\int R\, \sqrt{-g}\, d^4x$ can function as the gravitational action I_G.

However, as noted by Einstein himself, one can modify I_G as

$$I_G = \frac{-1}{16\pi G} \int (R + 2\lambda)\, \sqrt{-g}\, d^4x \tag{1.22}$$

without violating the requirements on the action as described above, where λ is some new constant. Einstein used a very small λ to obtain a stationary universe. This constant is known as the 'cosmological constant' since when it is small, it will not significantly affect the solutions, except in a cosmological context. When Einstein came to know about the observational evidence for the expansion of the universe, he decided to do away with it and described it as 'the greatest mistake in his life'. But this term λ is one of the most intriguing factors in current theoretical physics. It was later recognised that λ can also be a function of x^i [7].

With the introduction of $\lambda(x^i)$, the Einstein equation (1.20) can be written as

$$R^{ik} - \frac{1}{2} R g^{ik} - \lambda(x^i) g^{ik} = 8\pi G T^{ik}. \tag{1.23}$$

In view of its application in cosmology, the λ-term is usually taken to the right hand side of this equation, after making a substitution

$$\rho_\lambda = \frac{\lambda}{8\pi G},$$

so that

$$R^{ik} - \frac{1}{2}R\, g^{ik} = 8\pi G(T^{ik} + \rho_\lambda g^{ik}). \tag{1.24}$$

Using Eq. (1.19), one can see that the term $\rho_\lambda g^{ik}$ in the above equation is identical to the energy-momentum tensor for a perfect fluid having density ρ_λ and pressure $p_\lambda = -\rho_\lambda$.

1.1.3 Lagrangian Density

In the above subsection, we have seen that since the Ricci scalar R contains second derivatives of g_{ik} with respect to spacetime coordinates, the action will contain second derivatives. But in fact, an alternative expression for R, which does not contain any second derivatives of g_{ik} can be found [4] (and references therein) using the Arnowitt-Deser-Misner (ADM) 3+1 split of spacetime as

$$R = K^2 - K_{\mu\nu}K^{\mu\nu} - {}^3R. \tag{1.25}$$

This differs from the earlier expression (1.3) for R by a possible four-divergence. In the present case, we have conceived a foliation of spacetime into space-like hypersurfaces Σ_t labeled by t, which is some global time-like variable. ${}^3R$ is the scalar curvature of this 3-dimensional surface, $K_{\mu\nu}$ are the components of the extrinsic curvature of Σ_t defined by

$$K_{\mu\nu} = \frac{1}{2N}\left(N_{\mu|\kappa} + N_{\kappa|\mu} - \frac{\partial h_{\mu\nu}}{\partial t}\right) \tag{1.26}$$

and

$$K = h^{\mu\nu}K_{\mu\nu}. \tag{1.27}$$

N^μ is called the shift vector, N, the lapse function and $h_{\mu\nu} = n_\mu n_\nu - g_{\mu\nu}$ (where n_μ is the vector field normal to Σ_t) is the metric induced on this 3-space with $\sqrt{-g} = N\sqrt{h}$. " | " denotes covariant differentiation with respect to the spatial metric $h_{\mu\nu}$. The line element (1.2), in terms of the lapse N and shift N^μ is given as

$$ds^2 = g_{ik}dx^i dx^k = (N\ dt)^2 - h_{\mu\nu}(N^\mu dt + dx^\mu)(N^\nu dt + dx^\nu) \tag{1.28}$$

so that

$$g_{ik} = \begin{bmatrix} N^2 - N_\mu N_\nu h^{\mu\nu} & -N_\nu \\ -N_\mu & -h_{\mu\nu} \end{bmatrix} \tag{1.29}$$

and

$$g^{ik} = \begin{bmatrix} \frac{1}{N^2} & -\frac{N^\nu}{N^2} \\ -\frac{N^\mu}{N^2} & \frac{N^\mu N^\nu}{N^2} - h^{\mu\nu} \end{bmatrix} . \tag{1.30}$$

Thus the Lagrangian density to be used in the gravitational action I_G is

$$\mathcal{L}_G = -\sqrt{-g}R/16\pi G = -\frac{1}{16\pi G}\sqrt{h}N\left(K^2 - K_{\mu\nu}K^{\mu\nu} -{}^3 R\right). \tag{1.31}$$

The changes corresponding to that in the metric tensor are to be implemented in the matter action too. For example, in the case of a scalar field, Eq. (1.29) and (1.30) are to be used in the matter Lagrangian density

$$\mathcal{L}_\phi = \sqrt{-g}\Lambda_\phi = \sqrt{-g}\left[\frac{1}{2}g^{ik}\frac{\partial\phi}{\partial x^i}\frac{\partial\phi}{\partial x^k} - V(\phi)\right]. \tag{1.32}$$

One can write the Euler-Lagrange equations corresponding to variations with respect to N^μ, N and other dynamic variables in the total Lagrangian density $\mathcal{L}_G + \mathcal{L}_M = \mathcal{L}$. ($N^\mu$ and N are not dynamical variables; their time derivatives do not appear in $\mathcal{L}$. In fact, these are Lagrange multipliers so that after the variation one can fix some convenient gauge for them.) The equations obtained by varying with respect to N and N^μ are 'constraint equations' and they contain only first derivatives. Variation with respect to the other dynamical variables leads to field equations. The resulting equations can be seen to be the same as those obtained from the Einstein field equations (1.20). We shall make this explicit using specific examples in the next section.

1.2 Homogeneous and Isotropic Cosmologies

This section serves two purposes. First, it illustrates the formalism of GTR summarised in the last section by applying it to cosmology. But more importantly, it introduces the general framework of models which obey the cosmological principle [1]-[6]. Friedmann models form the basis of the standard hot big bang model whereas models with a minimally coupled scalar field paves the way for the inflationary cosmological models. We obtain the field equations for these models in the conventional way, but in the last subsection, we demonstrate their derivation using the Euler-Lagrange equations.

1.2.1 Friedmann Models

If the distribution of matter in space is homogeneous and isotropic, we can describe the spacetime by the maximally, spatially symmetric Robertson-Walker (RW) metric and obtain an important class of solutions to the Einstein field equations that are of much significance in cosmology. The RW line element is given by

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]. \tag{1.33}$$

$a(t)$ is the scale factor of the spatial expansion and $k = 0$, $+1$ or -1 which, in the respective order, corresponds to flat, positively curved or negatively curved spacelike hypersurfaces of constant t. Let us apply the formalism of GTR to this simple case.

Evaluating R and R_{ik} using (1.3) and (1.4), the Einstein equations (1.20) can be written for the perfect fluid described by (1.19) in a comoving frame with $U^i = (1, 0, 0, 0)$, which describes a homogeneous and isotropic distribution of matter as

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \tag{1.34}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi G p. \tag{1.35}$$

Differentiating (1.34) and combining with (1.35) gives

$$\frac{d(\rho a^3)}{da} + 3pa^2 = 0 \tag{1.36}$$

or equivalently

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p), \tag{1.37}$$

which is the conservation law for energy-momentum (1.21) in this case. Combining (1.34) and (1.35) in a different way, we get another useful result

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{1.38}$$

The solutions of these equations require, however, some additional information in the form of an 'equation of state' relating ρ and p. In most commonly encountered problems, we can write this relation as

$$p = w\rho. \tag{1.39}$$

It can be shown that for extreme relativistic matter, $w = 1/3$ and for nonrelativistic matter (dust), we have $w = 0$. These equations (1.34)-(1.39) were first obtained and studied by A. Friedmann and models based upon these are usually called Friedmann models. They predict either an expanding or contracting universe.

Eq. (1.36) can immediately be solved to obtain

$$\rho \propto a^{-3(1+w)}. \tag{1.40}$$

If there are more than one noninteracting component in ρ that are separately conserved, (1.36) and hence (1.40) are applicable to each. For relativistic matter, the density $\rho_{m,r} \propto a^{-4}$ and for nonrelativistic matter, $\rho_{m,nr} \propto a^{-3}$. The variation of ρ with a for other values of w can also be deduced from (1.40); for $w = -1/3$, $\rho \propto a^{-2}$ and for $w = -1$, ρ is a constant.

To study the variation of a with t, we make a few definitions. The quantity

$$H(t) \equiv \frac{\dot{a}}{a} \tag{1.41}$$

is called the Hubble parameter which measures the rate of expansion of the universe. The deceleration parameter $q(t)$ is defined through the relation

$$\frac{\ddot{a}}{a} \equiv -q(t)H^2(t) \tag{1.42}$$

and the critical density as

$$\rho_c \equiv \frac{3}{8\pi G}H^2. \tag{1.43}$$

Another important quantity is the density parameter

$$\Omega(t) = \frac{\rho}{\rho_c}. \tag{1.44}$$

Using these definitions, Eq. (1.34) can be written as

$$\Omega - 1 = \frac{k}{a^2H^2}. \tag{1.45}$$

The $k = 0$ case is a special one where $\Omega = 1$ or $\rho = \rho_c$. Using (1.40) in (1.34) gives the solution in this case as

$$a(t) \propto t^{2/3(1+w)}. \tag{1.46}$$

For the $k = +1$ case, $\Omega > 1$, $q > 1/2$ and the universe expands to a maximum and then recollapses. For $k = -1$, $\Omega < 1$, $q < 1/2$ and it expands for ever. The $k = 0$ case is critical in the sense that it just manages to expand for ever.

de Sitter Models

Instead of matter, if the RW spacetime contained only a cosmological constant, Eq. (1.24) (with $T^{ik} = 0$) leads to

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho_\lambda, \tag{1.47}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = 8\pi G\rho_\lambda. \tag{1.48}$$

The field equations are thus similar to a Friedmann model with equation of state $p_\lambda = -\rho_\lambda$; i.e., with $w = -1$. Thus a positive (negative) ρ_λ has a repulsive (attractive) effect so that we have an accelerating (decelerating) cosmic evolution with $\ddot{a} > 0$ ($\ddot{a} < 0$). It is the repulsive force due to a constant positive ρ_λ, which Einstein made use of in his

stationary universe model to prevent it from collapsing due to other matter distributions present.

Equations (1.47) and (1.48) are particularly simple to solve in the flat case with $k = 0$. The solution, with $H \equiv (8\pi G\rho_\lambda/3)^{1/2}$ = constant, is obtained as

$$a(t) \propto e^{Ht} \tag{1.49}$$

If we define

$$H = \sqrt{\frac{8\pi G\rho_\lambda}{3}}\ \tanh^k \left(\sqrt{\frac{8\pi G\rho_\lambda}{3}} t\right), \tag{1.50}$$

a solution can be found also for the $k = \pm 1$ cases. For $k = +1$,

$$a(t) \propto H^{-1}\ \cosh Ht \tag{1.51}$$

and for $k = -1$,

$$a(t) \propto H^{-1}\ \sinh Ht. \tag{1.52}$$

The model with positive ρ_λ is called the de Sitter model, after W. de Sitter, who solved it for the first time. The model with ρ_λ negative, is called the anti-de Sitter model.

1.2.2 Models With a Scalar Field

Another special case of interest is that of a RW spacetime filled with a minimally coupled scalar field ϕ, whose energy-momentum tensor is given by (1.17). With the assumption that ϕ is spatially homogeneous and depends only on time, Einstein equations can be written in a similar manner as that in (1.34)-(1.35)

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right], \tag{1.53}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi G\left[\frac{\dot{\phi}^2}{2} - V(\phi)\right]. \tag{1.54}$$

The equation of motion for ϕ can be obtained by using the conservation law for energy-momentum (1.21) as

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0. \tag{1.55}$$

It shall be noted that when the field is displaced from the minimum of its potential and when $\dot{\phi}^2 \ll V(\phi)$, Eq. (1.53) and (1.54) are similar to the Einstein equations (1.47) and (1.48), written for the spacetime containing only a cosmological constant (where we identify $V(\phi) = \rho_\lambda$). In this context, ρ_λ is usually called the vacuum energy

density. The solutions for spacetimes which contain a cosmological constant in addition to matter were studied by G. Lemaitre and such models are generally referred to as Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmologies.

1.2.3 Field Equations as Euler-Lagrange Equations

Lastly, let us demonstrate how the Einstein equations in different models are obtained as Euler-Lagrange equations under the variation of the action. For the RW spacetime, under the ADM 3+1 split, $N^\mu = 0$, $K_{\mu\nu} = -(1/N)(\dot{a}/a)h_{\mu\nu}$, ${}^3R = 6k/a^2$,

$$R = \frac{6}{N^2}\frac{\dot{a}^2}{a^2} - \frac{6k}{a^2} \tag{1.56}$$

and the action, using (1.31) and (1.32), is

$$\begin{aligned} I &= I_G + I_M = \int (\mathcal{L}_G + \mathcal{L}_M) d^4x \\ &= \int N\sqrt{h}\left[-\frac{1}{16\pi G}\left(\frac{6}{N^2}\frac{\dot{a}^2}{a^2} - \frac{6k}{a^2}\right) + \left(\frac{\dot{\phi}^2}{2N^2} - V(\phi)\right)\right] d^4x. \end{aligned} \tag{1.57}$$

Integrating the space part, we get

$$\begin{aligned} I &= 2\pi^2 \int N a^3 \left[-\frac{1}{16\pi G}\left(\frac{6}{N^2}\frac{\dot{a}^2}{a^2} - \frac{6k}{a^2}\right) + \frac{\dot{\phi}^2}{2N^2} - V(\phi)\right] dt \\ &\equiv \int L\, dt. \end{aligned} \tag{1.58}$$

Using the Lagrangian L, we may write the Euler-Lagrange equations for the variables N, a and ϕ and fixing the gauge $N = 1$ to obtain the same Einstein equations (1.53)-(1.55).

Similarly, for a de Sitter model which contains only a cosmological constant, the Lagrangian can be taken to be

$$L = 2\pi^2 N a^3 \left[-\frac{1}{16\pi G}\left(\frac{6}{N^2}\frac{\dot{a}^2}{a^2} - \frac{6k}{a^2}\right) - \rho_\lambda\right] \tag{1.59}$$

The Einstein equations (1.47) and (1.48) are obtained on writing the Euler-Lagrange equations corresponding to variations with respect to N and a, in the gauge $N = 1$.

1.3 The Standard Model - Its Successes

The standard model [2]-[6] claims to have the least amount of speculatory inputs into cosmology, while having maximum agreement with observations. It is based upon the following assumptions: (1) At the very large scales of the size greater than clusters of

clusters of galaxies, the universe is homogeneous and isotropic and hence is describable by the RW metric and (2) It is filled with relativistic/ nonrelativistic matter. Then the fundamental equations governing the evolution of the universe are those obtained earlier (1.34)-(1.39) with $w = 0$ or $1/3$. These models predict an expanding or contracting universe and belong to Friedmann cosmologies. We now discuss the three major success stories of the model, juxtaposing them with the current status of observational cosmology.

1.3.1 The Hubble Expansion

The 1929 discovery of a linear redshift-distance relation for galaxies by Hubble, if interpreted as due to Doppler effect, establishes the case for an expanding phase for the universe at present and was a primary piece of evidence in support of the standard model. At present, the expansion rate, characterised by the Hubble parameter (1.41) is in the range $H_p = 100\ h$ Km s^{-1} Mpc^{-1} ; $h = 0.7 \pm 0.05$. (The subscript p refers to the present epoch.) The Hubble radius $H_p^{-1} \approx 0.9 \times 10^{28} h^{-1}$ cm $\approx 2.9 \times 10^3 h^{-1}$ Mpc gives a measure of the size of the presently observed universe. The deceleration parameter defined by (1.42) is estimated to be lying in the range $-0.5 < q_p < 2$. Also the density parameter, as per current estimates is given by $0.1 \leq \Omega_p \leq 2$. The age of the universe, measured by direct observational dating techniques is $t_p \approx 5 \times 10^{17}$ s. Though these observations are not precise enough, they however confirm the Hubble expansion of the universe.

The observed redshift z of galaxies can be related to the scale factor a as

$$1 + z = \frac{a(t_p)}{a(t_1)} \tag{1.60}$$

where t_1 is the time at which the light is emitted. If we assume that the universe contains both radiation and matter, according to equation (1.40), before some time t_{eq} in its history, radiation will dominate over matter. In the standard cosmology, t_{eq} is estimated to be $\approx 1.35 \times 10^{11} \Omega^{-3/2} h^{-3}$ s. For a universe with flat space sections (i.e., $k = 0$), (1.46) gives $a \propto t^{1/2}$ for the relativistic era and $a \propto t^{2/3}$ for the nonrelativistic era. Assuming that the changeover is instantaneous, we can write

$$a = At^{2/3}, \qquad t > t_{eq} \tag{1.61}$$

$$a = Bt^{1/2}, \qquad t < t_{eq}. \tag{1.62}$$

Matching the two relations at $t = t_{eq}$, one estimates

$$\frac{B}{A} = \frac{t_{eq}^{2/3}}{t_{eq}^{1/2}} \approx 0.7 \times 10^2 \Omega^{-1/4} h^{-1/2} \mathrm{s}^{1/6}. \tag{1.63}$$

This value will be of use in evaluating expressions of the type (1.60) in the standard flat models. In both the other cases with $k = \pm 1$, we can regard the universe as nearly

flat when a was smaller than a_p by a few orders of magnitude (See flatness problem: Sec. 2.1).

1.3.2 Cosmic Microwave Background Radiation

Another important milestone in the development of the standard model was the discovery of the cosmic microwave background radiation (CMBR) by Penzias and Wilson in 1965. The spectrum of CMBR is consistent with that of a blackbody at temperature $T_p \approx 2.73K$. It endorses the view that there was a more contracted state for the universe, which ought to have been denser and hotter than the present. According to the standard model, the universe cools as it expands and when the temperature reaches $T \approx 4000K$, matter ceases to be ionised, the electrons join the atoms. Radiation is then no more in thermal equilibrium with matter (matter-radiation decoupling) and the opacity of the radiation drops sharply. The radiation we see now as CMBR is conceived as the relic of that last scattered at the time of decoupling. In fact, the CMBR was predicted by Gamow in 1948 and its discovery, perhaps, is the strongest observational evidence in support of the standard model.

We can derive an expression for the total relativistic matter (radiation) density $\rho_{m,r}$ in terms of temperature by the following argument [4]. (We use conventional units in this subsection.) For an ideal gas, there are $1/h^3$ number of states located in unit volume of μ-space, where h is the Planck's constant. The number of states in volume V with momentum less than P will be $(4/3)\pi P^3 V/h^3$. The occupancy of a single state is

$$\frac{1}{e^{(E_A(P)-\mu_A)/kT_A} \pm 1}.$$

$+(-)$ signs correspond to Fermi (Bose) statistics, μ_A is the chemical potential and T_A is the temperature of the species A which is assumed to be in equilibrium and $E_A(P) = (P^2c^2 + m^2c^4)^{1/2}$, the energy of a particle in the species A. Then the number of particles of type A with momentum between P and $P + dP$ per unit volume of space is

$$n_A(P)dP = \frac{g_A}{2\pi^2\hbar^3}\frac{P^2 dP}{e^{(E_A(P)-\mu_A)/kT_A} \pm 1}, \tag{1.64}$$

where g_A is the number of spin degrees of freedom. In the extreme relativistic ($T_A \gg m_A$) and nondegenerate ($T_A \gg \mu_A$) limit, the energy density, which corresponds to species A is

$$\rho_A = \int_0^\infty E_A(P)n_A(P)dP = g_A\sigma T_A^4 \qquad \text{(Bosons)} \tag{1.65}$$

$$= (7/8)g_A\sigma T_A^4 \qquad \text{(Fermions)} \tag{1.66}$$

where $\sigma = \pi^2 k^4/30\hbar^3 c^3 = 3.782 \times 10^{-15}$ erg m^{-3} K^{-4}. The total energy density contributed by all the relativistic species together can be written as

$$\rho_{m,r} c^2 = g_{tot} \sigma T^4, \tag{1.67}$$

where

$$g_{tot} = \sum_{(A=Bosons)} g_A (T_A/T)^4 + \sum_{(A=Fermions)} (7/8) g_A (T_A/T)^4 \tag{1.68}$$

is the effective number of spin degrees of freedom at temperature T. In the very early universe, g_{tot} is evaluated to be nearly equal to 100.

The expression for $\rho_{m,r}$ as given by (1.67) is a reasonable speculation if we agree to look upon the CMBR as the relic of a hot early universe. To obtain another useful result in the study of the thermal history of an expanding universe, we apply the second law of thermodynamics, in its familiar form, to a physical volume $V = a^3$;

$$kT\ dS = dE + pdV = d(\rho c^2 a^3) + pd(a^3) \tag{1.69}$$

and also use (1.36), which is a statement of the first law of thermodynamics. It is easy to see that

$$\frac{dS}{dt} = \frac{1}{kT}\left[\frac{d}{dt}(\rho c^2 a^3) + p\frac{d}{dt}(a^3)\right] = 0. \tag{1.70}$$

This implies that the entropy per comoving volume element of unit coordinate volume $V = a^3$, under thermal equilibrium, is a constant. i.e.,

$$S = \frac{(\rho c^2 + p)}{kT} a^3 = \text{constant}. \tag{1.71}$$

Thus in the standard model, the universe expands adiabatically. Eq. (1.67) implies that for radiation with $\rho_{m,r} \propto a^{-4}$, aT is a constant. In the relativistic era, for a $k = 0$ universe, this may be used to write

$$t = \left(\frac{3c^2}{32\pi G\sigma}\right)^{1/2} g_{tot}^{-1/2} T^{-2}. \tag{1.72}$$

The times at which radiation reaches various temperatures can be evaluated using this expression.

1.3.3 Primordial Nucleosynthesis

The third important success of the standard model is the prediction of primordial nucleosynthesis [3]-[6]. According to this theory, when the age of the universe was of the order of 1 s, the temperature was of the order of 10^{10} K and the conditions were right for nuclear reactions which ultimately led to the synthesis of significant amounts of D,

^{3}He, ^{4}He and ^{7}Li. The yields of these light elements, according to the model, depends on the baryon to photon ratio η and the number of very light particle species, usually quantified as the equivalent number of light neutrino species N_ν. The predictions of the abundance of the above four light elements agree with the observational data provided the free parameters η and N_ν in the theory have values in the range

$$2.5 \times 10^{-10} \leq \eta \leq 6 \times 10^{-10}, \qquad N_\nu. \leq 3.9 \tag{1.73}$$

In turn, if we accept the present abundance of light nuclei, the density parameter for baryons Ω_B may be predicted from the above to be lying in the range

$$0.01 \leq \Omega_B \leq 0.15, \tag{1.74}$$

which agrees with measured values. Furthermore, the bounds on N_ν

$$N_\nu = 3.0 \pm 0.02 \tag{1.75}$$

agree with particle accelerator experiments.

Chapter 2

Problems and Solutions

2.1 The Standard Model - Problems

The three major observational facts, namely, a linear redshift-distance relation, a perfect blackbody distribution for CMBR which corresponds to a more or less uniform temperature and the observed abundance of light elements have clearly established a case in favour of the standard, hot big bang model. However, this is only a broad brush picture and there are several loose ends to be sorted out when we go into details. There are issues like the formation of structures etc., which call for refinements of the theory. But here we focus attention on another class of puzzles, usually called 'cosmological problems', which deserve special attention since they indicate the possible existence of some inconsistencies in the standard model and hence do require substantial modifications in its underlying postulates. The most serious among them are the following.

Singularity Problem

The assumptions in the standard model (See Sec. 1.3) are in tune with the validity of the strong energy condition $\rho + 3p \geq 0$ and $\rho + p \geq 0$. This, when combined with some topological assumptions and causality conditions lead to strong singularity theorems which imply that a singularity, where the geometry itself breaks down, is unavoidable. In the cosmological context, this singularity corresponds to the instant of creation, the big bang, where quantities like matter density, temperature, etc., take unbounded values. The universe comes into existence at this instant, violating the law of conservation of energy, which is one of the most cherished principles of physics [8]. This is called the singularity problem.

Flatness Problem

From equation (1.45), which may be written in the form

$$\Omega - 1 = \frac{1}{\frac{8\pi G}{3}\frac{\rho a^2}{k} - 1}, \tag{2.1}$$

it is easy to see that for Ω being close to unity, $|\Omega - 1|$ grows as a^2 during the radiation dominated era ($\rho \propto a^{-4}$) and as a in the matter dominated era ($\rho \propto a^{-3}$). Thus since $\Omega(t_p)$ is still of the order of unity, at early times it was equal to 1, to a very high precision. For instance

$$\Omega(10^{-43}\ \mathrm{s}) = 1 \pm O(10^{-57}),$$

$$\Omega(1\ \mathrm{s}) = 1 \pm O(10^{-16}).$$

This means, for example, that if Ω at the Planck time $t_{pl} = 5.4 \times 10^{-44}$ s was slightly greater than 1, say $\Omega(10^{-43)}$ s$= 1 + 10^{-55}$, the universe would have collapsed millions of years ago. The standard model cannot explain why the universe was created with such fine-tuned closeness to $\Omega = 1$ [9]. This is the flatness problem.

Horizon Problem

The CMBR is known to be isotropic with a high degree of precision. Two microwave or infrared antennas pointed in opposite directions in the sky do collect thermal radiation with $\Delta T/T \leq 10^{-5}$, T being the black body temperature. In the context of the standard model, this is puzzling since these two regions from which CMBR of strikingly uniform temperature is emitted cannot have been in causal contact at any time in the past [10]. The problem can be explicitly stated as follows. According to the standard model, the proper distance to the horizon of the presently observed universe is of the order of H_p^{-1}. Since distances scale as $a(t)$, at any time in the past, say t_s, the size of the same part of the universe was $[a(t_s)/a(t_p)]H_p^{-1}$. But the distance a light signal can travel by the time t_s is equal to the proper distance to the horizon at that time; i.e.,

$$d_{hor}(t_s) = a(t_s)\int_0^{t_s}\frac{dt}{a(t)}. \tag{2.2}$$

If the presently observed part of the universe was to be in causal contact at t_s, a necessary (though not sufficient) condition is

$$d_{hor}(t_s) > \frac{a(t_s)}{a(t_p)}H_p^{-1}. \tag{2.3}$$

The isotropy of the CMBR, which was traveling unobstructed since the time of decoupling (t_{dec}), indicates that the presently observed part of the universe was in causal contact at least by that time. Hence, one would expect the above condition to be satisfied for some time $t_s < t_{dec}$. In the standard model, $t_{dec} \approx 10^{13}$ s and the time at which the universe changes from relativistic to nonrelativistic era is $t_{eq} \approx 10^{11}$ s. Using these

and also some typical values $\Omega = 1$, $h = 3/4$ and $t_p = 5 \times 10^{17}$ s, Eqs. (1.61)-(1.63) will help us to evaluate both sides of condition (2.3). It can be seen that the right hand side of this condition is greater than the left by a factor of $2.5 \times 10^7/t_s^{1/2}$ for $t_s < t_{eq}$ and by a factor of $0.63 \times 10^6/[(t_{eq}^{1/2}/40) + 3t_s^{1/3} - 3t_{eq}^{1/3}]$ for $t_s > t_{eq}$, thus violating the condition. For $t_s = t_{eq}$, this ratio is approximately equal to 80 and for $t_s = t_{dec}$, it is ≈ 10. This means that the presently observed part of the universe was not even in causal contact at the time of decoupling. Yet the surface of last scattering of radiation appears very much isotropic. This is the horizon problem.

Further, for the successful prediction of the primordial nucleosynthesis, the universe has to be homogeneous at least as early as ≈ 1 s. The condition (2.3) is then violated by a very wide margin.

Problem of Small Scale Inhomogeneity

The assumption in the standard model that the universe is homogeneous and isotropic is justifiable at least in the early epochs, before the matter-radiation equality. The remarkably uniform temperature of CMBR on all angular scales upto quadrupole, is ample evidence for this. But recent measurements show anisotropies (of the order of 10^{-5} or so) in CMBR in a systematic way and these anisotropies directly sample irregularities in the distribution of matter at the time of last scattering. It is believed that once the universe becomes matter dominated, small density inhomogeneities grow via the Jeans instability. Density inhomogeneities are usually expressed in a Fourier expansion

$$\frac{\delta\rho(\vec{x})}{\rho} = \frac{1}{(2\pi)^3} \int \delta_k \exp(-i\vec{k}.\vec{x}) d^3k, \tag{2.4}$$

where ρ is the mean density of the universe, $\vec{k}$ is the comoving wave number associated with a given mode and δ_k is its amplitude. So long as a density perturbation is of small magnitude(i.e., $\delta\rho/\rho \ll 1$), its physical wave number and wave length scale with $a(t)$ as $k_{phys} = k/a(t)$, $\lambda_{phys} = a(t) \times 2\pi/k$. Once a perturbation becomes nonlinear, it separates from the general expansion and maintains an approximately constant physical size. The inhomogeneity at present is: stars ($\delta\rho/\rho \approx 10^{30}$), galaxies ($\delta\rho/\rho \approx 10^5$), clusters of galaxies ($\delta\rho/\rho \approx 10 - 10^3$), superclusters or clusters of clusters of galaxies ($\delta\rho/\rho \approx 1$) and so on. Based upon this fact that nonlinear structures exist today, and the fact that in the linear regime fluctuations grow as $a(t)$ in the matter dominated epoch, we can calculate the amplitude of perturbations that existed on these scales at the epoch of decoupling. It should be possible to account for the anisotropies in the CMBR detected by the COBE satellite on this basis. The problem with this scenario of small scale inhomogeneity is that in the standard model, last scattering occurred at a redshift of around 1000 with the Hubble radius at that time subtending an angle of only around 1^o, while CMBR shows anisotropies on all angular scales upto the quadrupole [4].

This problem is closely related to the horizon problem in that if one imagines causal, microphysical processes acting during the earliest moments of the universe and giving rise to primeval density perturbations, the existence of particle horizons in the standard cosmology precludes production of inhomogeneities on the scales of interest.

Problem of the Size of the Universe

If we follow the standard evolution, the size of the comoving volume corresponding to the present Hubble volume at the Planck time t_{pl} can be evaluated using (1.61)-(1.63) as 10^{-4} cm. This is much greater than the Planck length $l_{pl} = 1.616 \times 10^{-33}$ cm, the only natural length scale available. This is the problem of the size of the universe [11].

Entropy Problem

Most of the entropy in the universe exists in the form of relativistic matter. The radiation entropy in the present Hubble volume may be evaluated using (1.67) in (1.71) as

$$S_p = \left(\frac{\rho_{m,r} + p_r}{T}\right)_p \frac{4}{3}\pi (H_p^{-1})^3 \approx 10^{88}, \tag{2.5}$$

with $g_{tot.} \approx 2$ and $T_p = 2.73$ K. The entropy at t_{pl}, again if we follow the standard evolution, will be the same as S_p. Where do such large numbers come from, is the entropy problem [11].

Monopole Problem

This is another problem closely related to the horizon problem. The Grand Unified Theories (GUTs) predict that as the universe cools down and the temperature reaches a value $\approx 10^{28}$ K, a spontaneous symmetry breaking occurs and as a result, magnetic monopoles are copiously produced. However, no such monopoles have yet been detected. This is the monopole problem [12].

The monopoles which are expected to be produced are of mass $\approx 2 \times 10^{-8}$ g. At the end of the GUT epoch (t_c), we expect at least one monopole per horizon size sphere to be produced. The horizon radius at t_c is given by $2t_c$. The radius of the same part of the universe at present is $d_{hor}(t_c)a(t_p)/a(t_c)$. Thus the present number density of monopoles will be of order

$$n_{monopole}(t_p) \approx \frac{1}{\frac{4\pi}{3}\left[d_{hor}(t_c)\frac{a(t_p)}{a(t_c)}\right]^3}. \tag{2.6}$$

With $T(t_c) \approx 10^{28}$ K and $d_{hor}(t_c) = 2t_c \approx 10^{-26}$cm, we can evaluate (with $a \propto T^{-1}$ and $T_p = 2.73$ K),

$$n_{monopole}(t_p) \approx 10^{-6}\text{cm}^{-3}. \tag{2.7}$$

With the monopole mass $\approx 2 \times 10^{-8}$ g, we get

$$\rho_{monopole}(t_p) \approx 2 \times 10^{-14} \text{g cm}^{-3}. \tag{2.8}$$

This is much greater than the closure density $\approx 10^{-29}$ g cm^{-3} of the present universe and if it were true, the universe would have collapsed much earlier. If the horizon problem is solved before t_c, the same dynamical mechanism would solve the monopole problem too. Further, this problem is not related to cosmology alone; if particle physics turns out to be discarding the hypothesis regarding monopole production, this problem will also disappear.

Cosmological Constant Problem

If we assume that the universe contains a cosmological constant $\rho_\lambda \equiv \lambda/8\pi G$ in addition to matter, then by measuring the values of the Hubble parameter $H = \dot{a}/a$ and the deceleration parameter $q = -\ddot{a}/aH^2$ and using the Einstein equations, one can find the magnitude of ρ_λ. Current estimates [13, 14] of this value is of the order of the critical density $\approx 10^{-29}$ g cm^{-3}. If ρ_λ is viewed as arising from the potential energy of a scalar field employed in field theoretic models, then no known symmetry principle in quantum field theory requires that its value be so small like this when compared to the Planck density $\rho_{pl} = 0.5 \times 10^{94}$ g cm^{-3}. That is, the measured value of ρ_λ is smaller than the expected value ρ_{pl} by 122 orders of magnitude. This is the cosmological constant problem [15].

2.2 Attempts to Modify the Standard Model

Let us now extrapolate backward in time the standard evolution for a universe with flat space using $\Omega = 1$, $h = 0.75$, $t_p = 5 \times 10^{17}$s and accepting the standard model values $z_{dec} \approx 1000$, $z_{eq} \approx 13500$ and $T_{nuc} \approx 10^{10}$ K (the subscripts *dec* and *nuc* refers, respectively, to the decoupling and nucleosynthesis epochs). We also normalise $a(t)$ such that $a(t_p)$ is equal to unity. Using the relation $1 + z = a(t_p)/a(t)$, we get

$$a_{dec} \approx 10^{-3}. \tag{2.9}$$

Writing $a = At^{2/3}$ in the matter dominated era, we can evaluate $t_{dec} \approx 1.58 \times 10^{13}$ s. Similarly, with $a = Bt^{1/2}$ in the relativistic era, we get $a_{eq} = 8.18 \times 10^{-5}$ and $t_{eq} = 3.18 \times 10^{11}$ s. Using $a \propto T^{-1}$ where T is the radiation temperature, we can evaluate $a_{nuc} = 3 \times 10^{-10}$ and $t_{nuc} = 5.2$ s. If we extrapolate still backward with the same expression $a = Bt^{1/2}$ till the Planck era, when the energy density is $\rho_{pl} = 0.5 \times 10^{94}$ g cm^{-3}, we get $a(\rho = \rho_{pl}) = 2.155 \times 10^{-32}$ and $t(\rho = \rho_{pl}) = 2.7 \times 10^{-44}$ s $\approx t_{pl}$. As mentioned in Sec. 2.1, this value of a corresponds to 10^{-4} cm for a region of size 10^{28} cm at present and is very large when compared to Planck length.

The horizon problem and the problem of generation of density perturbations above the present Hubble radius can be better understood in this context. We have already

stated that to account for the remarkable isotropy of CMBR, the condition is (2.3) (with $a_p = 1$): i.e.,

$$d_{hor}(t_{dec}) > a_{dec} H_p^{-1}, \tag{2.10}$$

where $d_{hor}(t_{dec})$ is given by equation (2.3). This statement may be extended to explain the anisotropies which correspond to the aforementioned density perturbations. Let us define

$$d_{comm.}(t_1, t_2) = \int_{t_1}^{t_2} \frac{dt}{a(t)}, \tag{2.11}$$

which is the communication distance light has traveled between times t_1 and t_2, evaluated at present. Then the condition for generation of density perturbations above the Hubble radius can be expressed as

$$d_{comm.}(t_{pl}, t_{dec}) > H_p^{-1}. \tag{2.12}$$

This is identical to the condition (2.3), provided we extend the lower limit of integration in (2.2) to t_{pl}. In the above scenario of standard evolution, the left hand side of Eq. (2.12) may be evaluated as equal to 1.1×10^{27} cm whereas the right hand side is approximately 1.2×10^{28} cm. This is an alternative way of stating the puzzles in the standard model.

Based on the behaviour of the scale factor alone, it was recently argued [16, 17] that some nonstandard evolution is essential for the solution of these problems. The argument is based upon the understanding that the standard model gives a reliable and tested accounting of the evolution of the universe, at least from the time of nucleosynthesis onwards. Hence it was asserted that if we do not want to jeopardise the successes of the standard model, the evolution should be standard, as described above, from $t \approx 1$ s onwards. Then one has to face the question of how to maximize (2.11), so that (2.12) is satisfied. Those models which do not violate the condition $\rho + 3p \geq 0$ has $\ddot{a} \leq 0$. In this scenario, (2.11) can be maximized by assuming a coasting evolution $\ddot{a} = 0$ or $a \propto t$. More specifically, Liddle [17] assumes

$$a(t) = \frac{a_{nuc}}{t_{nuc}} t, \qquad a < a_{nuc}. \tag{2.13}$$

In this picture, at the epoch when $\rho = \rho_{pl}$, we have $t \approx 3.75 \times 10^{-22}$ s. Between this epoch and that of nucleosynthesis, the maximum possible communication distance will be $\approx 3.55 \times 10^{22}$ cm, which is only a small fraction of $d_{comm}(t_{nuc}, t_{dec}) \approx 10^{27}$ cm. Thus the above problems cannot be solved in this picture. One has to conceive some nonstandard evolution characterised by $\ddot{a} > 0$ or $\rho + 3p < 0$. But this will necessitate some drastic modification to the standard model in that the existence of some kind of energy density with equation of state $p = w\rho$, $w < 0$ should be accepted, at least in the early epochs. One such case is a universe filled with the potential energy of a scalar

field. The resulting evolution, which resembles that of de Sitter cosmologies is called 'inflation'.

2.3 Inflation

It is well known how inflation solves the cosmological problems [18]. In all inflationary models, the universe which emerges from the Planck epoch, after a brief period of standard evolution (or sometimes without it) finds itself containing the potential energy of a scalar field, which is generally called the inflaton field. This field is initially displaced from the minimum of its potential and it rolls down slowly to that minimum. All viable inflationary models are of this slow rollover type or can be recast as such. Then the governing equations are (1.53)-(1.55) with $\dot{\phi}^2 \ll V(\phi)$ so that during this phase, the universe expands quasiexponentially as in (1.49) with H remaining a constant. That the inclusion of a minimally coupled scalar field will lead to such a dynamics was known much earlier. It was Guth [19] who showed that this phenomenon can possibly lead to the solution of cosmological problems. He showed that this exponential expansion stretches causally connected regions of size H^{-1} by an amount $\exp(H\Delta t)$ and consequently regions of size $H^{-1} \approx l_{pl}$ reach a size $H^{-1}\exp H\Delta t \approx 10^{-4}$ cm by the time inflation ends, provided $H\Delta t \approx 67$. This will help the universe to evolve as per the standard model for the rest of the time. This also will resolve the horizon problem. From (2.1), it is seen that during this period when $\rho_\phi \equiv V(\phi) \approx$ constant, for a closed universe, $\Omega - 1$ decreases as a^{-2} and by the end of the inflationary era, Ω can be arbitrarily close to unity. Similar is the behaviour of an open universe. Thus one need not start with any fine tuned initial conditions at the Planck epoch to get a nearly flat universe at present. This solves the flatness problem.

The inflationary stage gives way to the standard evolution when the scalar field reaches its minimum. During this process, the entropy of the universe increases enormously. This will solve the entropy problem. The monopole problem disappears along with the horizon problem.

The above are features generic to all inflationary models. But for the successful implementation of the mechanism, one has to decide on what type of field to constitute the inflaton field, what the potential of the field is and what initial conditions are to be specified. There are numerous inflationary models which differ in these matters. Guth [19] proposed his model as a possible solution to horizon, flatness and monopole problems in which the grand unified models tend to provide phase transitions that lead to an inflationary scenario of the universe. A grand unified model begins with a simple gauge group G which is a valid symmetry at the highest energies. As the energy is lowered, the theory undergoes a hierarchy of spontaneous symmetry breaking transitions into successive subgroups. At high temperatures, the Higgs fields of any spontaneously broken gauge theory would loose their expectation values, resulting in a high temperature phase in which the full gauge symmetry is restored. The effective potential $V(\phi, T)$ of the scalar field ϕ has a deep local minimum at $\phi = 0$, even at a

very low temperature T. As a result, the universe remains in a supercooled vacuum state $\phi = 0$, which is a false vacuum, for a long time. The energy-momentum tensor of such a state would be the same as that in equations (1.53)-(1.54) with $\dot{\phi}^2 \gg V(\phi)$ and the universe expands exponentially until the false vacuum decays. This phenomenon is termed a first order phase transition, which occurs at some critical temperature T_c. As the universe cools through this temperature, one would expect bubbles of the low temperature phase to nucleate and grow and these bubbles contain the field ϕ_0, which corresponds to the minimum of the effective potential $V(\phi)$. The universe will cool as it expands and it will then supercool in the high temperature phase. When the phase transition finally takes place at this low temperature, the latent heat is released and the universe is reheated to a temperature comparable to T_c. If the universe supercools to 28 or more orders of magnitude, sufficient entropy will be generated due to bubble wall collisions and thermalisation of energy.

As pointed out by Guth himself, the major problem with this scenario is that if the rate of bubble nucleation is greater than the speed of expansion of the universe, then the phase transition occurs very rapidly and inflation does not take place. On the other hand, if the vacuum decay rate is small, then the bubbles cannot collide and the universe becomes unacceptably inhomogeneous.

In order to improve this scenario, Linde [20] and, Albrecht and Steinhardt [21] independently suggested the 'new inflationary model'. The crucial difference between the new and old inflationary models is in the choice of the effective potential $V(\phi, T)$ and that the latter is a second order spontaneous symmetry breaking phenomenon. The new choice was the Coleman-Weinberg potential which has a bump near $\phi = 0$. For the decay from the false vacuum to the true vacuum, the system has to tunnel across the bump and then it slowly rolls down the potential. After reaching the minimum of the potential, it executes damped oscillations, during which energy is thermalised and entropy is increased. In the new inflation, a typical size of the bubble at the moment of its creation is $\approx 10^{-20}$ cm. After the exponential expansion, the bubble will have a size much greater than the observable part of the universe, so that we see no inhomogeneities caused by the wall collisions. The drawback of the new inflationary model is that it requires fine tuned initial values for the field.

Whereas the 'old' and 'new' inflationary models are the result of spontaneous symmetry breaking, the chaotic inflation [22] proposed by Linde does not contain any phase transition at all. The scalar field is not part of any unified theory and its only purpose is to implement inflation. The potential in chaotic inflation is assumed to be of the simple form

$$V(\phi) = \frac{l}{n}\phi^n. \tag{2.14}$$

The minimum of the potential is at $\phi = 0$ and so it has nothing to do with spontaneous symmetry breaking. With sufficiently large initial value, the field ϕ may roll slowly so that $a(t)$ rapidly approaches the asymptotic regime

$$a(t) = a_0 \exp\left[\frac{4\pi}{n}\left(\phi_0^2 - \phi^2(t)\right)\right]. \tag{2.15}$$

Linde envisions that the initial distribution of ϕ_0 is chaotic', with different values in different regions of the universe. In the $n = 4$ case, to obtain sufficient inflation, say 60 e-folds, ϕ_0 must be greater than about 4.4 m_{pl} where m_{pl} is the Planck energy. The model in its simplicity is not definite enough to discuss reheating.

A major drawback of chaotic inflation is the required smoothness of the initial inflationary patch. For inflation to occur, we see the condition

$$(\bigtriangledown\phi)^2 \ll \frac{l}{4}\phi_0^4. \tag{2.16}$$

If we take the dimension of the region over which ϕ varies by the order of unity to be L, then

$$(\bigtriangledown\phi)^2 \approx \left(\frac{\phi_0^2}{L}\right)^2 \ll \frac{l}{4}\phi_0^4 \tag{2.17}$$

which implies $L \gg (\phi_0/m_{pl})H^{-1}$. This means that, for sufficient inflation, ϕ must be smooth on a scale much greater than the Hubble radius, a condition which does not sound very chaotic.

In addition to these most widely discussed ones, there are many other models which exhibit inflation, but having a variety of features [18] (and references therein). The 'natural inflation' model is the one having the potential $V = V_0 \cos^2(\phi/m)$. Those models named 'power law inflation' have either $a \propto t^p$, $p > 1$ with potential $V(\phi) = V_0 \exp(-\mu\phi)$ or $a \propto (t_c - t)^{-q}$, $q > 1$ which obeys the induced gravity action, a variant of GR. Other models which make use of non-Einstein theories of gravity like the latter one are 'Starobinski model' (higher derivative gravity), 'Kaluza-Klein inflation' (higher dimensional Kaluza-Klein theories), 'extended inflation' (Brans-Dicke theory), 'pre-big bang inflation' (superstring theories) etc.. This shows that even after the two decades since the development of the theory of inflation, it still lacks a unique model.

Age and Other Problems

The most important prediction of the inflationary models is that the universe is almost spatially flat with the present value of the density parameter Ω_p very close to unity. This in turn implies that the combination $H_p t_p \approx 2/3$. A major set back to inflationary models was in fact this prediction. Recent observations [23] put this value to be lying in the range $0.85 < H_p t_p < 1.91$, contrary to the above prediction. This is the so called 'age problem' in the standard flat and inflationary models.

A way out for these models from the age problem is to postulate that there is a nonzero relic cosmological constant in the present universe, whose density parameter Ω_λ is comparable to that of matter. But since a cosmological constant is indistinguishable

from the vacuum energy which inflates the universe, the model is bound to explain how the enormous vacuum energy, which was present in the early universe gave way to such a small value at present. This of course will require another extreme fine tuning and is against the spirit of inflationary models, which were originally conceived to get rid of all sorts of fine tuning in the standard model.

This problem is further highlighted in the context of some very recent measurements [13] of the deceleration parameter, which indicates that in the present universe, q_p can even be negative. This can be interpreted as occurring due to the presence of a nonzero cosmological constant, whose density is comparable to that of the matter density. Thus, along with the age problem, the cosmological constant problem is aggravated by the inflationary models.

Lastly, the singularity problem is not addressed in the inflationary models. In most models, the inflationary stage is expected to occur at a time many orders of magnitude greater than the Planck time. Thus questions like how the universe came into 'existence' etc. are not addressed in the model.

2.4 Decaying-λ Cosmologies

The cosmological constant problem has triggered a lot of work in the literature [25]-[44] aimed at a solution based upon a dynamical λ; i.e., λ or equivalently $\rho_\lambda \equiv \lambda/8\pi G$ varying with time. An important motivation for considering a variable ρ_λ can be explained as follows [24]: If ρ_λ corresponds to the vacuum energy density, then its value is expected to be of the order of Planck density $\rho_{pl} = 0.5 \times 10^{94}$ g cm^{-3} at least in the Planck epoch. But all observations at present indicate a very low value $\approx 10^{-29}$ g cm^{-3} for this quantity. An order of magnitude calculation reveals that since the present age of the universe is $\approx 10^{61}$ times the Planck time, if ρ_λ decays with time from this initial value, then $\rho_{\lambda,p} \approx 0.5 \times 10^{94}/(10^{61})^2 \approx 10^{-29}$ g cm^{-3} as expected; i.e., the cosmological constant obeys an inverse square law in time.

All decaying-λ models are not precisely of this type. Here we discuss two pioneering decaying-λ cosmological models [25, 32] which propose phenomenological laws for the time-dependence of ρ_λ. It is of interest to note that while analysing the thermodynamic correctness of some decaying-λ models using the Landau-Lifshitz theory for non-equilibrium fluctuations, Pavon [34] has found that only these two models successfully pass their test. Special cases of these models are obtainable as the new cosmological model to be discussed in the following chapters of this thesis, which is derived at a more fundamental level.

2.4.1 Ozer and Taha Model

In the first of its kind, Ozer and Taha considered a decaying $-\lambda$ model [25] in which $\tilde{\rho} = \rho_m + \rho_\lambda$ with ρ_m denoting either the relativistic or nonrelativistic matter. The Einstein equation and the conservation equation in this case are

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_m + \rho_\lambda) \tag{2.18}$$

and

$$\frac{d}{dt}(\rho_m a^3) + p_m \frac{da^3}{dt} + a^3 \frac{d\rho_\lambda}{dt} = 0. \tag{2.19}$$

They noted that, for the solution of the cosmological problems, there should be entropy production and this will require $d\rho_\lambda/dt < 0$. Also they argued that, since the present matter density in the universe is close to the critical density and since these two are time-dependent terms in the fundamental dynamical equations of GTR, the equality of ρ_m and ρ_c would bestow some special status to the present epoch $t = t_p$. Thus they assume that this equality is a time-independent feature and impose the condition $\rho_m = \rho_c$ in the above equations. These conditions immediately yield

$$k = +1 \tag{2.20}$$

and

$$\rho_\lambda = \frac{3}{8\pi G}\frac{1}{a^2}. \tag{2.21}$$

In the relativistic era in which $p_{m,r} = \frac{1}{3}\rho_{m,r}$, they obtain a nonsingular solution

$$a^2 = a_0^2 + t^2, \tag{2.22}$$

where a_0 is the minimum value of the scale factor. The relativistic matter density is

$$\begin{aligned} \rho_{m,r} = \rho_0\left(\frac{a_0}{a}\right)^4 - \frac{1}{a^4}\int_{a_0}^{a} a'^4 \frac{d\rho_\lambda}{da'} da' &= \frac{3}{8\pi G}\left(\frac{1}{a^2} - \frac{a_0^2}{a^4}\right) \\ &= \frac{3}{8\pi G}\frac{t^2}{(a_0^2 + t^2)^2}. \end{aligned} \tag{2.23}$$

The radiation temperature T is assumed to be related to $\rho_{m,r}$ by the relation (1.67)

$$\rho_{m,r} = g_{tot}\sigma T^4, \tag{2.24}$$

from which

$$T = \left(\frac{3}{8\pi G g_{tot}\sigma}\right)^{1/4}\left[\frac{t^2}{(a_0^2 + t^2)^2}\right]^{1/4}. \tag{2.25}$$

Thus $T = 0$ at $t = 0$. The model predicts creation of matter at the expense of vacuum energy. A maximum temperature T_{max} is attained at $t = a_0$ and is given by

$$T_{max} = \left(\frac{3c^4}{8\pi G g_{tot} \sigma} \frac{1}{2a_0^2} \right)^{1/4}. \tag{2.26}$$

It was observed that T_{max} should correspond to the only energy scale present in the theory, which is the Planck energy and that this will give $a_0 \approx 0.03\ l_{pl}$ (assuming $g_{tot} \approx 100$ in the early relativistic era). Also for $t \gg a_0$, the values of the energy density and temperature attained by radiation at time t in the standard model are attained at time $2t$ in their model. Thus it is anticipated that the model has the same thermal history as the standard model. However, there is difference in the behaviour of the scale factor and also there is entropy production, which will help to solve the main cosmological problems. In particular, they have shown that causality will be established within a time $t_{caus} \approx 2.3a_0$, soon after the Planck epoch. It was also shown that the present monopole density in their model is much smaller than the critical density, which solves the monopole problem.

Though solutions are obtained for the pure radiation era with the above assumptions, they had to impose extra assumptions to determine the model when nonrelativistic matter is present. It was assumed that the early pure radiation era soon gave way to a period of matter generation, where $a_1 \le a \le a_2$. After that epoch, i.e., when $a \ge a_2$, $\rho = \rho_{m,r} + \rho_{m,nr}$ and Eq. (2.19) can be written as

$$d(\rho_m a^3) + d(\rho_{m,r} a^3) + p_{m,r}\ da^3 = -a^3 d\rho_\lambda. \tag{2.27}$$

In this era, it is assumed that

$$\mid \frac{d}{dt}(\rho_{m,nr} a_0^3 \mid \ll \mid \frac{d}{dt}(\rho_{m,r} a^3) \mid. \tag{2.28}$$

so that one obtains the solution

$$\rho_{m,r} = \frac{3}{8\pi G} \left(\frac{1}{a^2} + \omega \frac{a_p^2}{a^4} \right), \tag{2.29}$$

where ω is a dimensionless constant and a_p is the present value of the scale factor. Here, $\rho_{m,nr} a^3$, the total rest mass energy remains a constant. The regions $a_0 < a < a_1$ where $p_{m,r} = (1/3)\rho_{m,r}$ is separated from the regions $a \ge a_2$ by the epoch of matter creation, which may be considered as a region of phase transition. The time corresponding to a_1 is expected to be $t_1 \le 10^{-34}$ s; i.e., the GUT era. The reversal of sign of $\ddot{a}$ occurs during this time.

Throughout the evolution, the expression for p_λ is the same. Some predictions of the model are independent of the dimensionless parameter ω. These include $a_p \ge 1.578 \times 10^{30}$ cm, $\rho_\lambda \approx 8.26 \times 10^{-30}$ g cm^{-3}, $t_p \approx (2/3)H_p^{-1}$ and $q_p \approx 1/2$. Thus the values of t_p and q_p are nearly the same as those of the standard flat model.

It was observed by the authors themselves that the imposition of the condition $\rho_m = \rho_c$ is unphysical and that it may be worthwhile to seek a dynamic principle that

determines the form of ρ_λ, this being the most fundamental assumption made in the model.

2.4.2 Chen and Wu Model

Chen and Wu [32], while introducing their widely discussed decaying-λ cosmological model, have made an interesting argument in favour of an a^{-2} variation of the effective cosmological constant on the basis of some dimensional considerations in line with quantum cosmology. Their reasoning runs as follows: Since there is no other fundamental energy scale available, one can always write ρ_λ, the energy density corresponding to the effective cosmological constant as the Planck density ($\rho_{pl} = c^5/\hbar G^2 = 5.158 \times 10^{93}$ g cm^{-3}) times a dimensionless product of quantities. Assuming that ρ_λ varies as a power of the scale factor a, the natural ansatz is

$$\rho_\lambda \propto \frac{c^5}{\hbar G^2}\left(\frac{l_{pl}}{a}\right)^n \tag{2.30}$$

One can now show that $n = 2$ is a preferred choice. It is easy to verify that $n < 2$ (or $n > 2$) will lead to a negative (positive) power of $\hbar$ appearing explicitly in the right hand side of the above equation. Such an $\hbar$-dependent ρ_λ would be quite unnatural in the classical Einstein equation for cosmology much later than the Planck time. But it may be noted that $n = 2$ is just right to survive the semiclassical limit $\hbar \to 0$. This choice is further substantiated by noting that $n \leq 1$ or $n \geq 3$ would lead to a value of ρ_λ which violates the observational bounds. Thus Chen and Wu make the ansatz

$$\rho_\lambda = \frac{\gamma}{8\pi G a^2}, \tag{2.31}$$

where γ is a phenomenological constant parameter. Assuming that only the total energy-momentum is conserved, they obtain, for the relativistic era,

$$\rho_{m,r} = \frac{A_1}{a^4} + \frac{\gamma}{8\pi G a^2} \tag{2.32}$$

and for the nonrelativistic era,

$$\rho_{m,nr} = \frac{A_2}{a^3} + \frac{2\gamma}{8\pi G a^2} \tag{2.33}$$

where A_1 and A_2 are to be positive. The Chen-Wu model thus differs from the standard model in that it has a decaying cosmological constant and that the matter density has conserving and nonconserving parts [given by the first and second terms respectively in equations (2.32) and (2.33)]. By choosing γ appropriately, they hope to arrange ρ_λ and the nonconserving parts in $\rho_{m,r}$ and $\rho_{m,nr}$ to be insignificant in the early universe, so that the standard model results like nucleosynthesis are undisturbed. But for the late universe, it can have many positive features like providing the missing energy density in the flat and inflationary models, etc.. The model predicts creation of matter, but the authors argue that the creation rate is small enough to be inaccessible to observations.

Conversely to the requirement that the nonconserving parts of matter density should be negligible in the early universe for standard model results to remain undisturbed, one can deduce that in this model, the standard model results are applicable to only the conserving part of matter density. The nonconserving part is, in fact, created in the late universe. Thus for the standard model results to be applicable to the present universe, the conserving part of the matter density should be substantial. This in turn will create some problem with observations. For example, let us assume that at present, the conserved part of the nonrelativistic matter density is equal to the nonconserved part. Since the vacuum density is only one-half the nonconserved part [see equations (2.31) and (2.33)], for a $k = 0$ universe, the deceleration parameter at present will be $q_0 = (\Omega_m/2) - \Omega_\Lambda = 0.2$. This is not compatible with the observations mentioned earlier [13].

To avoid the problems in the early universe, they have to assume the occurrence of inflation, which in turn is driven by the vacuum energy. But they apply their ansatz to the late-time vacuum energy density (which corresponds to the cosmological constant) and not to that during inflation. But the stress energy associated with the vacuum energy is identical to that of a cosmological constant and it is not clear how they distinguish them while applying their ansatz.

Chapter 3

The New Model

There are a number of instances of the use of complex numbers or complex analytic functions in GTR [46]. Many of these applications have a common element, namely the analytic continuation of a real analytic manifold (the spacetime) into the complex, producing a complex spacetime. One such complex coordinate transformation is the Wick rotation of the time-coordinate t in Minkowski metric to obtain a Euclidean metric. Similarly the transformation of one solution of Einstein equation into another by means of a complex switch of coordinates is well known. For example, open and closed Friedmann models, de Sitter and anti de Sitter spacetimes, Kerr and Schwarschild metrics etc. are related by complex substitutions [47]. The use of complex variables extends to more sophisticated ones like spin-coefficient formalism, Ashtekar formalism, Twistor theory etc. We present our model based upon the signature change of the metric from Lorentzian (+ - - -) to Euclidean (++++). A signature change in the early universe is a widely discuused idea in current literature. After briefly reviewing the same, we present our model, which has a direct bearing on many cosmological observations, a feature unparalleled in most other applications mentioned above. We discuss the physical model including its predictions and then show how the model is devoid of cosmological problems.

3.1 Signature Change

The Hartle-Hawking 'no boundary' condition [48] in quantum cosmology allows a change of signature in the Planck epoch, resulting in the origin of the universe in a regime where there is no time. (The spacetime metric is Euclidean, so that spacetime is purely spatial). Ellis *et al.* [49] investigated such a possibility in the classical solution of the Einstein field equations. They argue that the usual solutions of this equation with Lorentzian metric are not because it is demanded by the field equations, but rather because it is a condition we impose on the metric before we start looking for solutions. They obtain a classical signature change by replacing the squared lapse function $N^2(t)$ appearing in the metric in the ADM 3+1 split of RW spacetime (See Secs. 1.1, 1.2)

with ν and allowing it to be negative. During signature change, ν passes through the value zero (which is a form of singularity) and hence a crucial point is the matching conditions at the surface of change. Ellis *et al.* have claimed to obtain RW solutions of the classical Einstein equations where ν changes sign at some time t_0, with the condition that the matter density and pressure are finite and the 3-space metric $h_{\mu\nu}$ is regular, as the change of sign takes place. This condition is equivalent to requiring that the extrinsic curvature $K_{\mu\nu}$ is continuous at the surface of change. In another approach, Hayward [50] obtained signature changing solutions by requiring that at the surface of change, $K_{\mu\nu}$ should vanish. The issue of these 'junction conditions' is a matter of hot debate in the current literature.

Another development in connection with the signature of the metric was that made by Greensite [51], who proposed a dynamical origin for the Lorentzian signature. The idea is to generalize the concept of Wick rotation in path integral quantisation. Rather than viewing Wick rotation as a mathematical technique for the convergence of the path integral, the Wick angle θ is treated as dynamical degree of freedom. He claims to have obtained a relation between the dimension and signature of spacetime, which favour a Lorentzian signature for a 4-dimensional spacetime and explain the presence of the factor i in the path integral amplitude. As a more general approach to signature change, Hayward [52] extended the idea of Greensite and allowed the lapse function to be complex. This is claimed to yield a complex action that generates both the usual Lorentzian theory and its Riemannian analogue and allows a change of signature between the two.

3.2 Derivation of the New model

We obtain a signature changing RW solution by a different route than those mentioned above [53, 54]. If we make a substitution $a(t) \to \hat{a}(t) = a(t)e^{i\beta}$ in Eq. (1.33), then the spacetime has Lorentzian signature (+ - - -) when $\beta = \pm n\pi$, $(n = 0, 1, 2, ..)$, and has Riemannian signature (++++) when $\beta = \pm(2n+1)\pi/2$, $(n = 0, 1, 2, ..)$. Let the solution $a(t)$ be in the form $a_o e^{\alpha(t)}$. Then the above expression becomes $\hat{a}(t) = a_o e^{\alpha(t)+i\beta}$. We note that interesting physics appears if the time-dependence of the scale factor is shared also by β; i.e., $\beta = \beta(t)$, an assumption consistent with the homogeneity and isotropy conditions. Then the signature of the metric changes when β varies from $0 \to \pi/2$ etc. Our ansatz is to replace $a(t)$ in metric (1.33) with

$$\hat{a}(t) = a(t)e^{i\beta(t)} = a_o e^{\alpha(t)+i\beta(t)} \equiv x(t) + i\, y(t). \tag{3.1}$$

We further assume that this model of the universe with a complex scale factor is closed (i.e.,$k = +1$) and has a zero energy-momentum tensor (i.e., $I_M = 0$). Thus we start with a system obeying an action principle, where the action is given by

$$I = \frac{-1}{16\pi G} \int (-g)^{1/2} R \, d^4x. \tag{3.2}$$

Here

$$R = \frac{6}{N^2}\left(\frac{\dot{\hat{a}}}{\hat{a}}\right)^2 - \frac{6}{\hat{a}^2}, \tag{3.3}$$

an expression similar to (1.56). Using this and integrating the space part, we get Eq. (3.2) as

$$I = -\frac{3\pi}{4G}\int N\,\hat{a}^3\left[\frac{1}{N^2}\left(\frac{\dot{\hat{a}}}{\hat{a}}\right)^2 - \frac{1}{\hat{a}^2}\right]dt \tag{3.4}$$

Minimising this action with respect to variations of N and $\hat{a}$ and fixing the gauge $N = 1$, we get the constraint and the field equations

$$\left(\frac{\dot{\hat{a}}}{\hat{a}}\right)^2 + \frac{1}{\hat{a}^2} = 0 \tag{3.5}$$

and

$$2\frac{\ddot{\hat{a}}}{\hat{a}} + \left(\frac{\dot{\hat{a}}}{\hat{a}}\right)^2 + \frac{1}{\hat{a}^2} = 0. \tag{3.6}$$

respectively. With $\hat{a}(t) \equiv x(t) + i\ y(t)$ and x_0, y_0 constants, these equations may be solved to get

$$\hat{a}(t) = x_0 + i\ (y_0 \pm t). \tag{3.7}$$

We can choose the origin $t = 0$ such that $\hat{a}(0) = x_0$. Relabelling $x_0 \equiv a_0$, we get,

$$\hat{a}(t) = a_0 \pm i\ t. \tag{3.8}$$

This equation gives the contour of evolution of $\hat{a}(t)$ which is a straight line parallel to the imaginary axis. At $t = 0$, this leaves the signature of spacetime Lorentzian but as $t \to \infty$ it becomes almost Riemannian. This need not create any conceptual problem since here we are considering only an unperceived universe with zero energy-momentum tensor whose existence is our ansatz. (Simple physical intuition would give a signature 'Riemannian at early times and Lorentzian at late' if it was for the physical universe we live in with matter contained in it. But in the above, we have a signature change in the opposite manner for the unphysical universe devoid of matter and this need not contradict our physical intuition). The connection with a closed real or physical universe is obtained by noting from the above that

$$a^2(t) = \mid \hat{a}(t) \mid^2 = a_0^2 + t^2. \tag{3.9}$$

This is the same equation (2.22) which governs the evolution of scale factor in the relativistic era of the Ozer-Taha model [25]. But in that model, a_0 is undetermined;

as mentioned in Sec. 2.4, it is only speculated to be of the order of Planck length. In our case, a quantum cosmological treatment to follow in Sec. 4.5 reveals that $a_0 = \sqrt{2G/3\pi} \approx l_{pl}$.

3.3 The Real Universe

Separating the real and imaginary parts of (3.5) and (3.6) and combining them, one easily obtains the following relations:

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \dot{\beta}^2 + \frac{2}{a^2}\sin^2\beta, \tag{3.10}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = 3\left(\dot{\beta}^2 + \frac{2}{3a^2}\sin^2\beta\right), \tag{3.11}$$

$$\ddot{\beta} + 2\dot{\beta}\frac{\dot{a}}{a} = 0, \tag{3.12}$$

and

$$2\dot{\beta}\frac{\dot{a}}{a} = \frac{1}{a^2}\sin 2\beta. \tag{3.13}$$

Also from (3.1) and (3.8), we get

$$\beta(t) = \tan^{-1}(\frac{\pm t}{a_0}) \tag{3.14}$$

and

$$\dot{\beta}(t) = \frac{\pm a_0}{a^2(t)} = \frac{\pm\cos^2\beta}{a_0}. \tag{3.15}$$

With the help of the last two equations we observe that the real parts of (3.5) and (3.6) can be rewritten in terms of $a = a(t) =\mid \hat{a}(t) \mid$ as

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{2}{a^2} - \frac{a_0^2}{a^4} \tag{3.16}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{2}{a^2} + \frac{a_0^2}{a^4}, \tag{3.17}$$

whose solution is the same as that obtained in (3.9). We see that the real quantity $a(t)$ may be considered as the scale factor of a nonempty RW universe. Eqs. (3.16) and (3.17) are appropriate for a closed RW model with real scale factor a and with total energy density and total pressure given by [See Eqs. (1.34) and (1.35)],

$$\tilde{\rho} = \frac{3}{8\pi G}\left(\frac{2}{a^2} - \frac{a_0^2}{a^4}\right), \tag{3.18}$$

$$\tilde{p} = -\frac{1}{8\pi G}\left(\frac{2}{a^2} + \frac{a_0^2}{a^4}\right) \tag{3.19}$$

respectively, whose breakup can be performed in many ways.

Matter and Vacuum

First let us assume, as done in [25], that

$$\tilde{\rho} = \rho_m + \rho_\lambda, \tag{3.20}$$

$$\tilde{p} = p_m + p_\lambda. \tag{3.21}$$

We write the equations of state in the form

$$p_m = w\ \rho_m \tag{3.22}$$

and

$$p_\lambda = -\rho_\lambda. \tag{3.23}$$

Solving (3.18) and (3.19) using (3.20)-(3.23), one gets

$$\rho_m = \frac{4}{8\pi G(1+w)}\left(\frac{1}{a^2} - \frac{a_o^2}{a^4}\right), \tag{3.24}$$

$$\rho_\lambda = \frac{1}{8\pi G(1+w)}\left[\frac{2(1+3w)}{a^2} + \frac{a_0^2(1-3w)}{a^4}\right]. \tag{3.25}$$

For a relativistic matter dominated universe, the matter density and the vacuum density are

$$\rho_{m,r} = \frac{3}{8\pi G}\left(\frac{1}{a^2} - \frac{a_0^2}{a^4}\right), \tag{3.26}$$

$$\rho_{\lambda,r} = \frac{3}{8\pi G}\frac{1}{a^2}. \tag{3.27}$$

From (3.17), the critical density of the real universe is

$$\rho_c \equiv \frac{3}{8\pi G}H^2 = \frac{3}{8\pi G}\left(\frac{1}{a^2} - \frac{a_o^2}{a^4}\right), \tag{3.28}$$

where H , the value of the Hubble parameter is assumed to coincide with that predicted by the model. (We can see that this is indeed the case in the present epoch by evaluating the combination $H_p t_p \equiv \left[\frac{\dot{a}}{a}\right]_p t_p$ for $a_p \gg a_0$, which is found to be nearly equal to

unity.) Then the ratios of density to critical density for matter and vacuum energy in the relativistic era are

$$\Omega_{m,r} \equiv \frac{\rho_{m,r}}{\rho_c} = 1, \tag{3.29}$$

$$\Omega_{\lambda,r} \equiv \frac{\rho_{\lambda,r}}{\rho_C} \approx 1 \qquad \text{for} \qquad a(t) \gg a_0. \tag{3.30}$$

For a universe dominated by nonrelativistic matter, the condition $w = 0$ may be used in (3.24) and (3.25). In this case,

$$\Omega_{m,nr} = 4/3, \tag{3.31}$$

$$\Omega_{\lambda,nr} \approx 2/3 \qquad \text{for} \qquad a(t) \gg a_0 \tag{3.32}$$

It may be noted that (3.26) and (3.27) are the same expressions as those obtained in [25] and (3.29) is their ansatz. But the last two results for the nonrelativistic era are outside the scope of that model.

Matter, Vacuum and Negative Energy

In the above, we have assumed $\tilde{\rho} = \rho_m + \rho_\lambda$ following the example in [25]. But this splitup is in no way unique. Equation (3.24) gives $\rho_m = 0$ at $t = 0$. In order to avoid this less probable result, we assume that the term $-(3/8\pi G)(a_0^2/a^4)$ in $\tilde{\rho}$ is an energy density appropriate for negative energy relativistic particles. The pressure $p_\_$ corresponding to this negative energy density $\rho_\_$ is also negative. Negative energy densities in the universe were postulated earlier [55]. Such an assumption has the further advantage of making the expressions for ρ_m and ρ_λ far more simple and of conforming to the Chen and Wu [32] prescription of a pure a^{-2} variation of vacuum density (though the Chen-Wu arguments, with a_0 identified as the Planck length, are not against the form (3.25) for ρ_λ since the term which contains a_0^2/a^4 becomes negligibly small when compared to the a^{-2} contribution within a few Planck times). Thus we use a modified ansatz in this regard [instead of (3.20)- (3.23)],

$$\tilde{\rho} = \rho_m + \rho_\lambda + \rho_\_, \tag{3.33}$$

$$\tilde{p} = p_m + p_\lambda + p_\_, \tag{3.34}$$

$$p_m = w\ \rho_m, \tag{3.35}$$

$$p_\lambda = -\rho_\lambda, \tag{3.36}$$

$$p_\_ = \frac{1}{3}\rho_\_, \tag{3.37}$$

and

$$\rho_{\_} = -\frac{3}{8\pi G}\frac{a_o^2}{a^4} \tag{3.38}$$

and solving (3.18) and (3.19) with these choices, the results are,

$$\rho_m = \frac{4}{8\pi G(1+w)}\frac{1}{a^2}, \tag{3.39}$$

$$\rho_\lambda = \frac{2(1+3w)}{8\pi G(1+w)}\frac{1}{a^2} \tag{3.40}$$

so that

$$\begin{gathered} \Omega_{m,r} \approx 1 \qquad \Omega_{\lambda,r} \approx 1, \\ \Omega_{m,nr} \approx 4/3 \qquad \Omega_{\lambda,nr} \approx 2/3 \\ \Omega_{\_} \equiv \frac{\rho_{=}}{\rho_C} \ll 1 \end{gathered} \tag{3.41}$$

for $a(t) \gg a_0$. The predictions for Ω_m are marginal, though not ruled out by observations.

Matter, Vacuum Energy, Negative Energy and K-matter

Many authors [56, 57, 58] seriously consider the existence of a new form of matter in the universe (called K-matter [56] - perhaps a stable texture [57]) with the equation of state $p_K = -\frac{1}{3}\rho_K$ and which decreases as a^{-2}. This leads to the idea of a low density closed universe [57]. If we accept this as probable, the prediction for Ω_m will be well within the observed range of values. In this case we include a term $\frac{3}{8\pi G}\frac{K}{a^2}$ to the right hand side of (3.33) so that

$$\rho_m = \frac{2}{8\pi G}\frac{1}{(1+w)}\frac{(2-K)}{a^2} \tag{3.42}$$

and

$$\rho_\lambda = \frac{1}{8\pi G}\frac{(1+3w)}{(1+w)}\frac{(2-K)}{a^2} \tag{3.43}$$

For a typical value $K = 1$ [56], the predictions for $a \gg a_0$ are

$$\begin{gathered} \Omega_{m,r} \approx 1/2, \qquad \Omega_{\lambda,r} \approx 1/2, \\ \Omega_{m,nr} \approx 2/3, \qquad \Omega_{\lambda.nr} \approx 1/3, \\ \Omega_{\_} \ll 1, \qquad \Omega_K \approx 1 \end{gathered} \tag{3.44}$$

The model makes clear cut predictions regarding the total energy density $\tilde{\rho}$ and total pressure $\tilde{p}$ as given by (3.18) and (3.19) but the decomposition of these do not follow from any fundamental principles, except for those heuristic reasons we put forward. It is easy to see that the conservation law for total energy

$$\frac{d(a^3\tilde{\rho})}{dt} = -\tilde{p}\,\frac{da^3}{dt} \tag{3.45}$$

is obeyed, irrespective of the ansatze regarding the detailed structure of $\tilde{\rho}$.

3.4 Thermal Evolution and Solution of Cosmological Problems

One can see that the solution of cosmological problems mentioned in Sec. 2.1 does not significantly depend on the split up of $\tilde{\rho}$. Note that in all the above cases, Ω_m is time-independent when $a \gg a_0$. Not only Ω_m, but also all the density parameters including the total density parameter (which may be defined as $\tilde{\Omega} \equiv \tilde{\rho}/\rho_c$) are constants in time. This is not difficult to understand: for $a \gg a_0$, $\tilde{\rho}$ and all other densities vary as a^{-2}. This, when put in (2.1) tells us that $\tilde{\Omega}$ and all other density parameters are time-independent for large t. Thus there is no flatness problem in this model.

Another notable feature is that in all the above cases, we have $\rho_m/\rho_\lambda = 2$ in the nonrelativistic era. Thus the model predicts that the energy density corresponding to the cosmological constant is comparable with matter density and this solves the cosmological constant problem too. It can also be seen that according to the model, the observed universe characterised by the present Hubble radius has a size equal to the Planck length at the Planck epoch and this indicates that the problem with the size of the universe does not appear here.

Next let us consider the horizon problem. A necessary condition for the solution of this problem before some time t_s is given by Eq. (2.3). Using our expression (3.9) for the scale factor with $a_0 \approx l_{pl}$, we see that the horizon problem is solved immediately after the Planck epoch, even if we extend the lower limit of integration in (2.2) to t_{pl}. For the investigation of other problems, we have to study the thermal evolution of the universe as envisaged in the model.

The relativistic matter density in the present model [using (3.39) or more generally (3.42)] can be written as

$$\rho_{m,r} = \frac{3}{8\pi G}\frac{\kappa}{a^2} = \frac{3}{8\pi G}\frac{\kappa}{a_0^2+t^2}, \tag{3.46}$$

where $\kappa = 1 - \frac{K}{2}$ is a constant of the order of unity. Using (1.67) we find the corresponding temperature as

$$T = \left(\frac{3}{8\pi G}\frac{\kappa}{g_{tot}\sigma}\right)^{1/4}\left(\frac{1}{a_0^2+t^2}\right)^{1/4}, \tag{3.47}$$

which is a maximum at $t = 0$. (In natural units $\sigma = \pi^2/30$.) If $a_0 = \sqrt{2G/3\pi}$ as mentioned in Sec. 3.2, then $T(0) \approx 0.36 \times \kappa^{1/4}G^{-1/2}$, which is comparable with the Planck energy and as $t \to \infty$, T decreases monotonically.

The above expressions (3.46) and (3.47) may be compared with the corresponding expressions in the standard model:

$$\rho_{s.m.} = \frac{3}{8\pi G}\frac{1}{(2t)^2}, \tag{3.48}$$

$$T_{s.m.} = \left[\frac{3}{8\pi G}\frac{1}{g_{tot}\sigma}.\right]^{1/4}\frac{1}{(2t)^{1/2}} \tag{3.49}$$

Assuming that $\kappa^{1/4}$ is close to unity, it can be inferred that the values of $\rho_{m,r}$ and T attained at time t in the standard model are attained at time $\sqrt{2}t$ in the present model. Thus the thermal history in the present model is expected to be essentially the same as that in the standard model. But the time-dependence of the scale factor is different in our model; we have a nearly coasting evolution and this helps us to solve the cosmological problems.

It can now be shown that density perturbations on scales well above the present Hubble radius can be generated in this model by evaluating the communication distance light can travel between the Planck time t_{pl} and t_{dec}, the time of decoupling [17]:

$$d_{comm}(t_{pl}, t_{dec}) = a_p\int_{t_{pl}}^{t_{dec}}\frac{dt}{a(t)} = 0.62 \times 10^6\text{Mpc} \tag{3.50}$$

where we have used $t_{dec} \approx 10^{13}s$, the same as that in the standard model, an assumption which is justifiable on the basis of our reasoning made before regarding thermal history. Thus the evolution in our case has the communication distance between t_{pl} and t_{dec} much larger than the present Hubble radius and hence it can generate density perturbations on scales of that order. [See Eq. (2.12).] It is interestng to note that Liddle [17] has precluded coasting evolution as a viable means to produce such perturbations and argued that only inflation ($\ddot{a} > 0$) can perform this task, thus "closing the loopholes" in the arguments of Hu *et al.* [16]. But it is worthwhile to point out that his observations are true only in a model which coasts from t_{pl} to t_{nuc} and thereafter evolves according to the standard model (See Sec. 2.2). In our case, the evolution is coasting throughout the history of the universe (except during the Planck epoch) and hence his objection is not valid.

A bonus point of the present approach, when compared to standard and inflationary models may now be noted. In these models, the communication distance between t_{nuc} and t_{dec}, or for that matter the communication distance from any time after the production of particles (assuming this to occur at the end of inflation) to the time

t_{dec} will be only around $200h^{-1}$ Mpc [17]. Thus density perturbations on scales above the Hubble radius cannot be generated in these models in the period when matter is present. This is because inflation cannot enhance the communication distance after it. The only means to generate the observed density perturbations is then to resort to quantum fluctuations of the inflaton field. The present model is in a more advantageous position than the inflationary models in this regard since the communication distance between t_{nuc} and t_{dec} in this case is

$$d_{comm}(t_{nuc}, t_{dec}) = a_p \int_{t_{nuc}}^{t_{dec}} \frac{dt}{a(t)} = 4.35 \times 10^{29}\text{cm} = 1.45 \times 10^{5}\text{Mpc} \tag{3.51}$$

which is again much greater than the present Hubble radius. So we can consider the generation of the observed density perturbations as a late-time classical behaviour too.

It can be seen that entropy is produced at the rate

$$\frac{dS}{dt} = 4\pi^2 \frac{3\kappa}{8\pi G} \left[\frac{8\pi G}{3} \frac{g_{tot}\sigma}{\kappa}\right]^{1/4} \frac{t}{(a_0^2 + t^2)^{1/4}} \tag{3.52}$$

which enables the solution of cosmological problems.

Lastly, the present monopole density predicted in this case can be seen to be [See Eq. (2.6)]

$$n_{monopole}(t_P) \approx \frac{3}{4\pi} a^{-3}(t_p) \approx \frac{3}{4\pi} \times 10^{-84}\text{cm}^{-3} \tag{3.53}$$

so that

$$\rho_{monopole} \approx \frac{3}{2\pi} \times 10^{-92}\text{g cm}^{-3} \tag{3.54}$$

This is very close to that estimated in [25], and is negligibly smaller than the critical density. Thus the monopole problem is solved also in this case.

Irrespective of the case we are considering, the model is nonsingular and there is no singularity problem. The solution of the age problem is also generic to the model. It may be noted that the model correctly predicts the value of the combination $H_p t_p \approx 1$. This places the present theory in a more advantageous position than the standard flat and the inflationary models with a zero cosmological constant, where this value is predicted to be equal to $2/3$, which is not in the range of recently observed values.

Another interesting feature is that since the expansion process is reversible and the basic equations are time reversal invariant, we can extrapolate to $t < 0$. This yields an earlier contracting phase for the universe. Such a phase was proposed by Lifshitz and Khalatnikov [59]. If there was such an initial phase, causality could have established itself much earlier than the time predicted in [25].

The model predicts creation of matter at present with a rate of creation per unit volume given by

$$\frac{1}{a^3}\frac{d(a^3\rho_m)}{dt}\,|_p = \rho_{m,p}\,H_p, \tag{3.55}$$

where $\rho_{m,p}$ is the present matter density. In arriving at this result, we have made use of the assumption of a nonrelativistic matter dominated universe. Note that the creation rate is only one third of that in the steady-state cosmology [5]. Since the possibility of creation of matter or radiation at the required rate cannot be ruled out at the present level of observation [32], this does not pose any serious objection.

3.5 Alternative Approach

We present an alternative model to the above without resorting to any complex metric, while preserving all the positive features of the physical universe envisaged in it, except the avoidance of singularity. We do this by modifying the Chen-Wu argument (See Sec. 2.4) to include the conserved total energy density $\tilde{\rho}$ of the universe in place of the vacuum density and this again brings in some fundamental issues which need serious consideration. If the Chen-Wu ansatz is true for ρ_λ, then it should be true for $\tilde{\rho}$ too. In fact, this ansatz is better suited for $\tilde{\rho}$ rather than ρ_λ since the Planck era is characterised by the Planck density for the universe, above which quantum gravity effects become important. Hence one can generalise (2.30) to write

$$\tilde{\rho} = A\frac{c^5}{\hbar G^2}\left(\frac{l_{pl}}{a}\right)^n \tag{3.56}$$

where A is a dimensionless proportionality constant. When $\tilde{\rho}$ is the sum of various components and each component is assumed to vary as a power of the scale factor a, then the Chen-Wu argument can be applied to conclude that $n = 2$ is a preferred choice for each component. Violating this will force the inclusion of $\hbar$ -dependent terms in $\tilde{\rho}$, which would look unnatural in a classical theory. Not only for the Chen and Wu model, in all of Friedmann cosmologies, this argument may be used to forbid the inclusion of substantial energy densities which do not vary as a^{-2} in the classical epoch.

At first sight, this may appear as a grave negative result. But encouraged by our results in the previous sections, we proceed to the next logical step of investigating the implications of an a^{-2} variation of $\tilde{\rho}$. If the total pressure in the universe is denoted as $\tilde{p}$, the above result that the conserved quantity $\tilde{\rho}$ in the Friedmann model varies as a^{-2} implies $\tilde{\rho} + 3\tilde{p} = 0$. This will lead to a coasting cosmology. Components with such an equation of state are known to be strings or textures [57]. Though such models are considered in the literature, it would be unrealistic to consider our present universe as string dominated. A crucial observation which makes our model with $\tilde{\rho}$ varying as a^{-2} realistic is that this variation leads to string domination only if we assume $\tilde{\rho}$ to be unicomponent. Instead, if we assume that $\tilde{\rho}$ consists of parts corresponding to relativistic/ nonrelativistic matter and a time-varying cosmological constant, i.e., if we assume

$$\tilde{\rho} = \rho_m + \rho_\lambda, \qquad \tilde{p} = p_m + p_\lambda, \tag{3.57}$$

then the condition $\tilde{\rho} + 3\tilde{p} = 0$ will give

$$\frac{\rho_m}{\rho_\lambda} = \frac{2}{1+3w} \tag{3.58}$$

In other words, the modified Chen-Wu ansatz leads to the conclusion that if the universe contains matter and vacuum energies, then vacuum energy density should be comparable to matter density. This, of course, will again lead to a coasting cosmology, but a realistic one when compared to a string dominated universe.

ρ_m or ρ_λ, which varies as a^{-2}, may sometimes be mistaken for strings but it should be noted that the equations of state we assumed for these quantities are different from that for strings and are what they ought to be to correspond to matter density and vacuum energy density respectively. It is true that components with equations of state $p = w\ \rho$ should obey $\rho \propto a^{-3(1+w)}$, but this is valid when those components are separately conserved. In our case, we have only assumed that the total energy density is conserved and not the parts corresponding to ρ_m and ρ_λ separately. Hence there can be creation of matter from vacuum, but again the present creation rate is too small to make any observational consequences.

The solution to the Einstein equations in a Friedmann model with $\tilde{\rho} + 3\tilde{p} = 0$, for all the three cases $k = 0, \pm 1$, is the coasting evolution

$$a(t) = \pm mt \tag{3.59}$$

where m is some proportionality constant. The total energy density is then

$$\tilde{\rho} = \frac{3}{8\pi G}\frac{(m^2+k)}{a^2}. \tag{3.60}$$

Comparing this with (3.56) (with n=2), we get $m^2 + k = 8\pi A/3$.

The prediction regarding the age of the universe in the model is obvious from Eq. (3.59). Irrespective of the value of m, we get the combination $H_p t_p$ as equal to unity, which is well within the bounds. Thus there is no age problem in this model. We can legitimately define the critical density as $\rho_c \equiv (3/8\pi G)(\dot{a}^2/a^2)$, so that equation (3.60) gives

$$\tilde{\Omega} \equiv \frac{\tilde{\rho}}{\rho_c} = \left(1 - \frac{3k}{8\pi A}\right)^{-1} \tag{3.61}$$

As in the standard model, we have $\tilde{\Omega} = 1$ for $k = 0$ and $\tilde{\Omega} > 1$ (< 1) for $k = +1$ ($k = -1$). But unlike the standard model, $\tilde{\Omega}$ is a constant. Also for A greater than or approximately equal to 1, we have $\tilde{\Omega}$ close to unity for all values of k. Using equation (3.57) and (3.58), we get

$$\Omega_m \equiv \frac{\rho_m}{\rho_c} = \frac{2\tilde{\Omega}}{3(1+w)}, \qquad \Omega_\lambda \equiv \frac{\rho_\lambda}{\rho_c} = \frac{(1+3w)\tilde{\Omega}}{3(1+w)} \tag{3.62}$$

It is clear that we regain our model in the previous section when $m = 1$ and $k = +1$. In that special case, $A = 3/4\pi$ and the present alternative model is precisely the same as the former, except for the initial singularity and the evolution in the Planck epoch. But even when A is not exactly equal to $3/4\pi$ and is only of the order of unity, the thermal evolution is almost identical and the absence of cosmological problems is generic to these models.

Chapter 4

Quantum Cosmology

Among the fundamental interactions of nature, gravity stands alone; it is linked to geometry of spacetime by GTR while the other interactions are describable by quantum fields which propagate in a 'background spacetime'. Another reason is that whereas quantum field theory assumes a preferred time coordinate and a previlaged class of observers, GTR demands equivalence among all coordinate systems. Also in quantum theory, there is the issue of 'observation': the quantum system is supposed to interact with an external observer who is described by classical physics, but such notions are alien to GTR. To sum up, we can say that till now these two major physical theories remain disunited. Quantum gravity is an attempt to reconcile them. It is not yet clear what a quantum theory of gravity is, and there are several directions pursued in this regard. Perhaps the simplest application of quantum gravity is in cosmology. The most well studied approach in quantum cosmology is the canonical quantisation in which one writes a wave equation for the universe, analogous to the Schrodinger equation. This procedure requires a Hamiltonian formulation of GTR. In this chapter, we present a brief review of quantum cosmology and then apply the formalism to the cosmological models discussed in the last chapter.

4.1 Hamiltonian Formulation of GTR

In Sec. 1.1, we obtained the gravitational Lagrangian density as a function of N, N_μ and $h_{\mu\nu}$ as

$$\mathcal{L}(N, N_\mu, h_{\mu\nu}) = -\frac{\sqrt{h}N}{16\pi G}(K^2 - K_{\mu\nu}K^{\mu\nu} - {}^3R). \tag{4.1}$$

The extrinsic curvature $K_{\mu\nu}$ involves time derivatives of $h_{\mu\nu}$ and spatial derivatives of N_μ. The three-curvature ${}^3R$ involves only spatial derivatives of $h_{\mu\nu}$. Since the Lagrangian density does not contain time derivatives of N or N_μ, the momenta conjugate to N and N_μ vanish:

$$\pi \equiv \frac{\delta \mathcal{L}}{\delta \dot{N}} = 0, \tag{4.2}$$

$$\pi^{\mu} \equiv \frac{\delta \mathcal{L}}{\delta \dot{N}_{\mu}} = 0. \tag{4.3}$$

These expressions are called primary constraints. The momenta conjugate to $h_{\mu\nu}$ are

$$\pi^{\mu\nu} \equiv \frac{\delta \mathcal{L}}{\delta \dot{h}_{\mu\nu}} = \frac{\sqrt{h}}{16\pi G}(K^{\mu\nu} - h^{\mu\nu} K). \tag{4.4}$$

The gravitational canonical Hamiltonian for a closed geometry can now be formed as

$$\mathcal{H}_c = \int (\pi^{\mu\nu} \dot{h}_{\mu\nu} + \pi^{\mu} \dot{N}_{\mu} + \pi \dot{N} - \mathcal{L})\, d^3x. \tag{4.5}$$

As usual for the Hamiltonian theory, one removes $\dot{h}_{\mu\nu}$, $\dot{N}_{\mu}$ and $\dot{N}$ and express $\mathcal{H}_c$ in terms of the coordinates N, N^{μ} and $h^{\mu\nu}$ and the conjugate momenta π, π^{μ} and $\pi^{\mu\nu}$. Since the primary constraints $\pi = \pi^{\mu} = 0$ hold at all times, we have $\dot{\pi} = \dot{\pi}^{\mu} = 0$. Writing the Poisson brackets for $\dot{\pi}$ and $\dot{\pi}^{\mu}$, we find

$$\dot{\pi} = \{\mathcal{H}_c, \pi\} = \frac{\delta \mathcal{H}_c}{\delta N} = 0, \tag{4.6}$$

$$\dot{\pi}^{\mu} = \{\mathcal{H}_c, \pi^{\mu}\} = \frac{\delta \mathcal{H}_c}{\delta N_{\mu}} = 0. \tag{4.7}$$

When generalised to include the matter variables and their conjugate momenta, these expressions give the secondary constraints, which are formally equivalent, respectively, to the time-time and time-space components of the classical Einstein field equations.

The arena in which the classical dynamics takes place is called ‘superspace’, the space of all three-metrics and the matter field configurations on a three-surface [4]. This involves an infinite number of degrees of freedom and hence to make the problem tractable, all but a finite number of degrees of freedom must be frozen out. The resulting finite dimensional superspace is known as a ‘minisuperspace’. In the following, we consider a minisuperspace model in which the only degrees of freedom are those of the scale factor a of a closed RW spacetime and a spatially homogeneous scalar field ϕ. The Lagrangian for this problem is given by (1.58) with $k = +1$; i.e.,

$$L = -\frac{3\pi}{4G} N \left[\frac{a\dot{a}^2}{N^2} - a - \frac{8\pi G}{3} \left(\frac{\dot{\phi}^2}{2N^2} - V(\phi) \right) a^3 \right], \tag{4.8}$$

from which we find the conjugate momenta π_a and π_{ϕ} as

$$\pi_a = \frac{\partial L}{\partial \dot{a}} = -\frac{3\pi}{2G} \frac{a\dot{a}}{N} \tag{4.9}$$

and

$$\pi_\phi = \frac{\partial L}{\partial \dot{\phi}} = 2\pi^2 \frac{a^3 \dot{\phi}}{N}. \tag{4.10}$$

The canonical Hamiltonian can now be constructed as

$$\begin{aligned}
\mathcal{H}_c &= \pi_a \dot{a} + \pi_\phi \dot{\phi} - L \\
&= N \left[-\frac{G}{3\pi} \frac{\pi_a^2}{a} - \frac{3\pi}{4G} a + \frac{3\pi}{4G} a^3 \left(\frac{G}{3\pi^3} \frac{\pi_\phi^2}{a^6} + \frac{8\pi G}{3} V(\phi) \right) \right] \\
&\equiv N\mathcal{H}.
\end{aligned} \tag{4.11}$$

The secondary constraint (4.6) now give

$$\mathcal{H} = -\frac{G}{3\pi} \frac{\pi_a^2}{a} - \frac{3\pi}{4G} a + \frac{3\pi}{4G} a^3 \left(\frac{G}{3\pi^3} \frac{\pi_\phi^2}{a^6} + \frac{8\pi G}{3} V(\phi) \right) = 0, \tag{4.12}$$

which is equivalent to (1.53). For the RW spacetime which contains only a cosmological constant, (1.59) helps us to write the constraint equation as

$$\mathcal{H} = -\frac{G}{3\pi} \frac{\pi_a^2}{a} - \frac{3\pi}{4G} a + \frac{3\pi}{4G} a^3 \frac{8\pi G}{3} \rho_\lambda = 0. \tag{4.13}$$

This equation is equivalent to (1.47). In all cases, $\mathcal{H}$ is independent of the lapse N and shift N^μ and thus the latter quantities are Lagrange multipliers (as mentioned towards the end of Sec. 1.1) and not dynamical variables. Stated in a different way, the fact that $\mathcal{H} = 0$ is a consequence of a new symmetry of the theory, namely, time reparametrisation invariance. This means that using a new time variable t' such that $dt' = N\, dt$ will not affect the equations of motion. Also this enables one to choose some convenient gauge for N, a procedure we adopt on several occasions. The constraint equation gives the evolution of the true dynamical variable $h_{\mu\nu}$ (a in the above examples) and can be used in place of the Hamilton equations.

4.2 Wheeler-DeWitt Equation

Canonical quantisation of a classical system like the one above means introduction of a wave function $\Psi(h_{\mu\nu}, \phi)$ [4, 60, 61] and requiring that it satisfies

$$i\frac{\partial \Psi}{\partial t} = \mathcal{H}_c \Psi = N\mathcal{H}\Psi. \tag{4.14}$$

To ensure that time reparametrisation invariance is not lost at the quantum level, the conventional practice is to ask that the wave function is annihilated by the operator version of $\mathcal{H}$; i.e.,

$$\mathcal{H}\Psi = 0. \tag{4.15}$$

But some other authors [62] argue that by defining a new variable τ such that $N\ dt = d\tau$, one can retain the form (4.14); i.e.,

$$i\frac{\partial\Psi}{\partial\tau} = \mathcal{H}\Psi \tag{4.16}$$

and the resulting quantum theory will still be reparametrisation invariant. However, in the following we use the more conventional form (4.15), which is called the Wheeler-DeWitt (WD) equation.

This equation is analogous to a zero energy Schrodinger equation, in which the dynamical variables $h_{\mu\nu}$, ϕ etc. and their conjugate momenta $\pi_{\mu\nu}$, π_ϕ etc. (generally denoted as q^α and p^α, in the respective order) are replaced by the corresponding operators. The wave function Ψ is defined on the superspace and we expect it to provide information regarding the evolution of the universe. An intriguing fact here is that the wave function is independent of time; they are stationary solutions in the superspace. The wave functions commonly arising in quantum cosmology are of WKB form and may be broadly classified as oscillatory, of the form e^{iS} or exponential, of the form e^{-I}. The oscillatory wave function predicts a strong correlation between q^α and p^α in the form

$$p_\alpha = \frac{\partial S}{\partial q^\alpha}. \tag{4.17}$$

S is generally a solution to the Hamilton-Jacobi equations. Thus the wave function of the form e^{iS} is normally thought of as being peaked about a set of solutions to the classical equations and hence predicts classical behaviour. A wave function of the form e^{-I} predicts no correlation between coordinates and momenta and so cannot correspond to classical behaviour.

In a minisuperspace, one would expect Ψ to be strongly peaked around the trajectories identified by the classical solutions. But these solutions are subject to observational verification, at least in the late universe so that a subset of the general solution can be chosen as describing the late universe. Now the question is whether the solution to the WD equation can discern this subset too. But it shall be noted that, just like the Schrodinger equation, the WD equation merely evolves the wave function and there are many solutions to it. To pick one solution, the normal practice is to specify the initial quantum state (boundary condition). These boundary conditions, through the wave function, therefore set initial conditions for the solution of classical equations. Then one may ask whether or not the finer details of the universe we observe today are consequences of the chosen theory of initial conditions.

In the simple example of the RW spacetime which contain only a cosmological constant, Ψ is defined on the minisuperspace with one dimension in the variable a. We replace $\pi_a \to i\ d/da$ in (4.13) to write the WD equation as

$$\left[\frac{d^2}{da^2} - \frac{9\pi^2}{4G^2}\left(a^2 - \frac{8\pi G}{3}\rho_\lambda a^4\right)\right]\Psi(a) = 0. \tag{4.18}$$

The factor ordering in the operator replacement in (4.13) is ambigous. For many choices of factor ordering, the effect can be parametrised by a constant r and the corresponding Hamiltonian operator is obtained by the substitution

$$\pi_a^2 \to -a^{-r}\left(\frac{\partial}{\partial a}a^r\frac{\partial}{\partial a}\right). \tag{4.19}$$

The choice in (4.18) corresponds to $r = 0$. But it will not significantly affect the semiclassical calculations and hence we choose the form in (4.18) for convenience. In this form the WD equation resembles a one-dimensional Schrodinger equation written for a particle with zero total energy, moving in a potential

$$U(a) = \frac{9\pi^2}{4G^2}\left(a^2 - \frac{8\pi G}{3}a^4\rho_\lambda\right). \tag{4.20}$$

Let us now define

$$a_0 \equiv \left(\frac{8\pi G}{3}\rho_\lambda\right)^{-1/2}. \tag{4.21}$$

In the particle analogy, there is a forbidden region for the zero energy particle in the intervel $0 < a < a_0$ and a classically allowed region for $a > a_0$. The WKB solutions of (4.18) in the classically allowed region $a > a_0$ are

$$\Psi_\pm(a) = \pi_a^{-1/2}\exp\left[\pm i\int_{a_0}^{a}\pi_{a'}da' \mp i\pi/4\right], \tag{4.22}$$

where $\pi_a = [-U(a)]^{1/2}$. In the forbidden region the solutions are

$$\bar{\Psi}_\pm(a) =\mid \pi_a \mid^{-1/2}\exp\left[\pm\int_{a}^{a_0} \mid \pi_{a'} \mid da'\right]. \tag{4.23}$$

For $a \gg a_0$, we have

$$-i\frac{d}{da}\Psi_\pm(a) \approx \pm\pi_a\Psi_\pm(a). \tag{4.24}$$

Thus Ψ_- and Ψ_+ describe, respectively, an expanding and contracting universe. It is now that we impose boundary conditions and different boundary conditions lead to different predictions. Some of such well-motivated proposals for the boundary conditions are by Hartle-Hawking, Vilenkin and Linde [48, 63, 64].

4.3 Boundary Condition Proposals

The Hartle-Hawking 'no boundary' boundary condition [48] is expressed in terms of a Euclidean path integral. The corresponding wave functon, in the present case, is specified by requiring that it is given by $\exp(-I_E)$ in the under barrier regime, where I_E is the Euclidean action. This gives

$$\Psi_H(a < a_0) = \bar{\Psi}_-(a), \tag{4.25}$$

$$\Psi_H(a > a_0) = \Psi_+(a) - \Psi_-(a). \tag{4.26}$$

This corresponds to a real wave function with equal mixture of expanding and contracting solutions in the classically allowed region. Linde's wave function [63] is obtained by reversing the sign of the exponential in the Euclidean regime;

$$\Psi_L(a < a_0) = \bar{\Psi}_+(a), \tag{4.27}$$

$$\Psi_L(a > a_0) = \frac{1}{2}\left[\Psi_+(a) + \Psi_-(a)\right]. \tag{4.28}$$

Vilenkin's 'tunneling boundary condition' [64] gives a purely expanding solution for the classical regime

$$\Psi_T(a > a_0) = \Psi_-(a) \tag{4.29}$$

and the under-barrier wave function is

$$\Psi_T(a < a_0) = \bar{\Psi}_+(a) - \frac{i}{2}\bar{\Psi}_-(a). \tag{4.30}$$

The growing exponential $\bar{\Psi}_-(a)$ and the decreasing exponential $\bar{\Psi}_+(a)$ have comparable amplitudes at $a = a_0$, but away from that point the decreasing exponential dominates. This, he describes as creation of the universe from 'nothing'.

This quantisation scheme is applied to spacetimes which contain scalar fields. The attempt is to examine the possibility of emergence of a semiclassical phase from the quantum cosmological era, which contains a scalar field with the required initial conditions for inflation to occur. On using the Hartle-Hawking wave function, the probability for tunneling from $a = 0$ to $a = a_0$ is given by

$$P_H \propto e^{-I_E}. \tag{4.31}$$

Under the Vilenkin tunneling boundary condition,

$$P_L \propto e^{-|I_E|}. \tag{4.32}$$

If the potential of the scalar field has several extrema, then using the latter prescription, tunneling favours the maximum with largest value of $V(\phi)$ (which is advantageous for inflation) whereas the former prescription favours the minimum with the smallest value of the potential. However, all these authors agree that these proposals may be criticised on the grounds of lack of generality or lack of precision [60, 61].

4.4 Quantisation of the New Physical Models

Quantisation of the models discussed in Ch. 3 involves a slight paradigm shift: we do not have inflation and also our models always contain matter along with vacuum energy. Though our prototype model is the one with zero energy-momentum tensor and a complex scale factor, we postpone the discussion on that to the next section. First we consider our coasting model discussed in Sec. 3.5, with total energy density varying as a^{-2}.

We adopt the approach of Fil'chenkov [65], who has considered the WD equation for flat, closed and open universes which allow for some kind of matter other than vacuum. He generalises the potential $U(a)$ given by (4.20) for $\rho_\lambda=$ constant by writing the energy density for the universe in the form

$$\tilde{\rho} = \rho_{pl} \sum_{n=0}^{6} B_n \left(\frac{l_{pl}}{a}\right)^n . \tag{4.33}$$

Here $n = 3(1+w)$.This is a superposition of partial energy densities of various kinds of matter at Plankian densities, each one of them being separately conserved. The kinds of matter included are

$$\begin{array}{lllll} n &=& 0 & (w=-1) & \text{vacuum}, \\ n &=& 1 & (w=-2/3) & \text{domain walls}, \\ n &=& 2 & (w=-1/3) & \text{strings}, \\ n &=& 3 & (w=0) & \text{dust}, \\ n &=& 4 & (w=1/3) & \text{relativistic matter}, \\ n &=& 5 & (w=2/3) & \text{bosons and fermions}, \\ n &=& 6 & (w=1) & \text{ultra stiff matter}, \end{array} \tag{4.34}$$

The WD equation is now written as

$$\left[\frac{d^2}{da^2} - U(a)\right]\Psi = 0, \tag{4.35}$$

with the generalised form of the potential (4.20)

$$U(a) = \frac{9\pi^2}{4G^2}\left(ka^2 - \frac{8\pi G}{3}a^4\tilde{\rho}\right). \tag{4.36}$$

We too proceed along similar lines, but first considering only a single conserved component at a time. It shall be noted that the constraint (1.34) and field equations (1.35) for an energy density $\rho = C_n/a^n$ with equation of state (1.39) (where $w = \frac{n}{3} - 1$) are obtainable from the Lagrangian

$$L = 2\pi^2 a^3 N \left[-\frac{1}{16\pi G} \left(\frac{6}{N^2} \frac{\dot{a}^2}{a^2} - \frac{6k}{a^2} \right) - \frac{C_n}{a^n} \right] \tag{4.37}$$

by writing the Euler-Lagrange equation corresponding to variation with respect to N and a. The Hamiltonian is

$$\mathcal{H} = -\frac{G}{3\pi} \frac{\pi_a^2}{a} - \frac{3\pi}{4G} ka + \frac{3\pi}{4G} a^3 \frac{8\pi G}{3} \frac{C_n}{a^n} = 0 \tag{4.38}$$

and the WD equation in this case can be written as

$$\left[\frac{d^2}{da^2} - \frac{9\pi^2}{4G^2} \left(ka^2 - \frac{8\pi G}{3} a^{4-n} C_n \right) \right] \Psi_n(a) = 0. \tag{4.39}$$

For $n > 2$, classically there is a forbidden region for $a > a_0$, whereas the allowed region is for $a < a_0$; $a_0 \equiv [(8\pi G/3)C_n]^{1/(n-2)}$. We see that for the special case with $n = 2$, the WD equation reduces to

$$\left[\frac{d^2}{da^2} - \frac{9\pi^2}{4G^2} a^2 \left(k - \frac{8\pi G}{3} C_2 \right) \right] \Psi = 0. \tag{4.40}$$

With $C_2 = (3/8\pi G)(m^2 + k)$, this corresponds to the energy density (3.60) advocated by us. In this case, the WD equation is simply

$$\left[\frac{d^2}{da^2} + \frac{9\pi^2}{4G^2} a^2 m^2 \right] \Psi = 0. \tag{4.41}$$

It is clear that Ψ is oscillatory for all values of a. If we choose the factor ordering corresponding to $r = -1$ [instead of $r = 0$: See Eq. (4.19)] in the above, we have

$$\left[\frac{d^2}{da^2} - \frac{1}{a} \frac{d}{da} + \frac{9\pi^2}{4G^2} a^2 m^2 \right] \Psi = 0, \tag{4.42}$$

which has an exact solution

$$\Psi(a) \propto \exp \left(\pm i \frac{3\pi}{4G} m a^2 \right). \tag{4.43}$$

It is of interest to note that if we define $\Psi \equiv e^{iS}$ in the above, S satisfies the Hamilton-Jacobi equation

$$\left(\frac{dS}{da} \right)^2 + U(a) = 0, \tag{4.44}$$

where $U(a) = -(9\pi^2/4G^2)a^2 m^2$. The classical constraint in this case is $\pi_a^2 + U(a) = 0$. This invites the identification

$$\pi_a^2 = \left(\frac{dS}{da}\right)^2 = \frac{9\pi^2}{4G^2}a^2m^2. \tag{4.45}$$

Using this in our definition $\pi_a \equiv \partial L/\partial \dot{a} = -(3\pi/2G)a\dot{a}$ [as in (4.9), with $N = 1$], we get $\dot{a} = \pm m$, from which the coasting evolution is regained. Thus the oscillatory wave function is strongly peaked about the singular coasting evolution throughout the history of the universe.

Now let us turn to the physical universe with total energy density $\tilde{\rho}$ given by (3.18). Clearly the field equation and constraint (3.17) follow from the Lagrangian

$$L = \frac{3\pi}{4G}\left(-\frac{\dot{a}^2 a}{N^2} - a + \frac{a_0^2}{a}\right). \tag{4.46}$$

The Hamiltonian is

$$\mathcal{H} = -\frac{G}{3\pi}\frac{\pi_a^2}{a} + \frac{3\pi}{4G}\left(a - \frac{a_0^2}{a}\right) = 0, \tag{4.47}$$

so that the WD equation, with factor ordering $r = 0$, is

$$\left[\frac{d^2}{da^2} - \frac{9\pi^2}{4G^2}\left(a_0^2 - a^2\right)\right]\Psi = 0. \tag{4.48}$$

The potential in this case indicates that $a < a_0$ is a classically forbidden region. The classical action $\int L\, dt$ constructed using (4.46) in this under-barrier region can be seen to be

$$S = i\frac{3\pi}{2G}a_0^2\{\frac{1}{2}\cos^{-1}\left(\frac{a}{a_0}\right) - \frac{1}{4}\sin 2\left[\cos^{-1}\left(\frac{a}{a_0}\right)\right]\}, \tag{4.49}$$

which is pure imaginary. It can be seen that for $a \ll a_0$, e^{iS} satisfies the WD equation. Similarly for the region $a > a_0$, the classical action is evaluated as

$$S = \frac{3\pi}{2G}\left[\left(a^2 - a_0^2\right)^{1/2} a - a_0^2 \cosh^{-1}\left(\frac{a}{a_0}\right)\right] \tag{4.50}$$

which is real. Also in this case, e^{iS} is a solution for $a \gg a_0$. Using a reasoning like that in the case of (4.43), we can regain the solution (3.9) in both cases.

4.5 Quantisation of the Complex, Source-free Model

Lastly, we quantise the model with complex scale factor and zero energy-momentum tensor [54] and show that this model has the correct classical correspondence with the classical trajectory. From (3.4), the Lagrangian for the problem is obtained as $L = -(3\pi/4G)\left(\dot{\hat{a}}^2\hat{a} - \hat{a}\right)$. The conjugate momentum to $\hat{a}$ is

$$\pi_{\hat{a}} = \frac{\partial L}{\partial \dot{\hat{a}}} = -\frac{3\pi}{2G}\hat{a}\dot{\hat{a}}. \tag{4.51}$$

The Hamiltonian is

$$\mathcal{H} = -\frac{G}{3\pi}\frac{\pi_{\hat{a}}^2}{\hat{a}} - \frac{3\pi}{4G}\hat{a}. \tag{4.52}$$

The constraint equation $\mathcal{H} = 0$ has the corresponding WD equation

$$(\mathcal{H} - \epsilon)\Psi(\hat{a}) = 0 \tag{4.53}$$

where we have made a modification such that an arbitrary real constant ϵ is introduced to take account of a possible energy renormalisation in passing from the classical constraint to its quantum operator form, as done by Hartle and Hawking in [48]. It shall be noted that this equation is still reparametrisation invariant. Choosing the operator ordering for the sake of simplicity of the solution, we get,

$$\frac{d^2\Psi(\hat{a})}{d\hat{a}^2} - (\frac{9\pi^2}{4G^2}\hat{a}^2 + \frac{3\pi}{G}\epsilon\hat{a})\Psi(\hat{a}) = 0 \tag{4.54}$$

Making a substitution $\hat{S} = \sqrt{3\pi/2G}\,[\hat{a} + (2G/3\pi)\epsilon]$, this becomes,

$$\frac{d^2\Psi(\hat{S})}{d\hat{S}^2} + (\frac{2G}{3\pi}\epsilon^2 - \hat{S}^2)\Psi(\hat{S}) = 0. \tag{4.55}$$

The wave equation has ground state harmonic oscillator type solution for $\epsilon = \sqrt{3\pi/2G}$:

$$\Psi(\hat{a}) = \mathcal{N} \;\; \exp\left[-\frac{3\pi}{4G}\left(\hat{a} + \sqrt{\frac{2G}{3\pi}}\right)^2\right]. \tag{4.56}$$

This is nonnormalisable, but it is not normal in quantum cosmology to require that the wave function should be normalised [60]. Our choice is further justified by noting that the probability density

$$\Psi^\star\Psi = \mathcal{N}^2 \;\; \exp\left(\frac{3\pi}{2G}y^2\right) \; \exp\left[-\frac{3\pi}{2G}\left(x + \sqrt{\frac{2G}{3\pi}}\right)^2\right] \tag{4.57}$$

is sharply peaked about the classical contour given by Eq. (3.8), which is a straight line parallel to the imaginary axis with x remaining a constant. We can identify a_0 with the expectation value of x;

$$a_0 \equiv < x > = -\sqrt{\frac{2G}{3\pi}} \tag{4.58}$$

so that $\mid a_0 \mid \approx l_{pl}$, which is the desired result. The $\exp\left[(3\pi/4G)\, y^2\right]$ part of the wave function is characteristic of a Riemannian space-time with signature $(+\,+\,+\,+\,)$. This is precisely the feature we should expect to correspond to the imaginary part in the scale factor.

The classical correspondence can be made more explicit by making an argument similar to that made in Sec. 4.4. The classical Hamiltonian constraint equation in this case is

$$\pi_{\hat{a}}^2 + \frac{9\pi^2}{4G^2}\hat{a}^2 = 0. \tag{4.59}$$

Defining the above wave function (4.56) as $\Psi(\hat{a}) \equiv e^{iS}$, we note that S satisfies the Hamilton-Jacobi equation

$$\left(\frac{dS}{da}\right)^2 + \frac{9\pi^2}{4G^2}\left(\hat{a} + \sqrt{\frac{2G}{3\pi}}\right)^2 = 0 \tag{4.60}$$

Comparing these two equations, we see that e^{iS} is strongly peaked about the classical solution, for large $\hat{a}$ when compared to the Planck length.

Thus the result obtained on quantisation is that the simplest minimum energy wave function is sharply peaked about the classical contour of evolution of $\hat{a}$, just like the ground state harmonic oscillator wave function in quantum mechanics is peaked about the classical position of the particle. But we welcome the important difference with this analogy; ie., the quantum mechanical system in our case is not localised. In fact, the wave function is not normalisable along the imaginary axis. If it was with real scale factor, the exponential growth of the wave function would correspond to some classically forbidden region, but in this case, we have the nonnormalisable part for the wave function along the imaginary axis; this result is just what we should expect since it corresponds to our classical system and cannot be termed as 'classically forbidden'. The most significant fact is that the quantum cosmological treatment helps us to predict the value of a_0, the minimum radius in the nonsingular model as compared to the Planck length.

Chapter 5

Reprise

5.1 Comparison of Solutions

For the purpose of comparison with the solution of Einstein equations in the new cosmological model, we take a closer look at the occurrence of inflation mentioned in Sec. 2.3. In the scalar field model described by (1.53) -(1.55), we see that the field equations (1.54) and (1.55) are second order partial differential equations, whose solution involves initial values of four quantities a, $\dot{a}$, ϕ and $\dot{\phi}$. The constraint equation (1.53) connects first derivatives and hence the number of arbitrary parameters in the theory gets reduced to three. The occurrence of inflation in this system is not generic; it depends crucially on several factors [60]. First of all, the potential $V(\phi)$ should be of the inflationary type; i.e., that for some range of values of ϕ, $V(\phi)$ should be large and $\mid \frac{dV(\phi)}{d\phi}/V(\phi) \mid \ll 1$. For the subset of $k = +1$ solutions, inflation occurs only when the initial value of $\dot{\phi} \approx 0$. It is argued that quantum cosmology provides such initial conditions favourable for inflation to occur, by the choice of proper boundary conditions. In this case, the cosmological wave function is peaked around the trajectories defined by

$$\dot{a} \approx \left[\frac{8\pi G}{3} a^2 V(\phi)\right]^{1/2} \gg 1, \qquad \dot{\phi} \approx 0. \tag{5.1}$$

Then the number of free parameters are reduced to two. Integrating the above equations give

$$a \approx \exp\left[\sqrt{\frac{8\pi G}{3}} V^{1/2}(t - t_0)\right], \qquad \phi \approx \phi_0 = \text{constant} \tag{5.2}$$

Here t_0 and ϕ_0 are the two arbitrary constants parametrising this set of solutions. The constant t_0 is in fact irrelevant, because it is the origin of the unobservable parameter time. However, this solution is inflationary.

Let us contrast this situation with the solution of the system described by (3.5) and (3.6). The complex field equation (3.6) is in fact a set of two second order partial

differential equations and involves four free parameters. But also the constraint equation contains two first order equations:

$$\dot{x} = 0, \qquad \dot{y} = \pm 1 \tag{5.3}$$

This helps us to obtain the desired solution $\hat{a} = a_0 \pm it$, which corresponds to the nonsingular physical model, as a general one and without resorting to quantum cosmology. Since t_0 is irrelevant, a_0 is effectively the only free parameter in the classical theory. Quantum cosmology, in fact, allows us to predict this value too, as comparable to Planck length. This prediction is not in the way a_0 is identified in the conventional quantum cosmological theories mentioned in Sec. 4.3 or in our own models discussed in Sec. 4.4. In these cases, a_0 can be readoff from the potential itself and is not obtained as an expectation value using the wave function.

Another feature that distinguishes our quantum cosmological treatment is that we are not imposing any adhoc boundary conditions; we only look for an exact solution to the WD equation. This procedure is quite similar to the solution of harmonic oscillator problem in ordinary quantum mechanics. In this sense, introduction of a zero point energy in the WD equation (4.53) is justifiable. However, we adopt the point of view that the vanishing of the classical Hamiltonian can be taken care of by restricting the solution to that corresponding to the minimum energy. It is of interest to note that this minimum (zero point) energy is $\epsilon = (3\pi/2)^{1/2}\epsilon_{pl}$ where ϵ_{pl} is the Planck energy. That is, the total energy of the universe is not zero; it is the Planck energy - apart from a numerical factor.

At this point, it is worth while to point out that the total positive energy (matter, vacuum etc.) contained in the closed real universe at $t = 0$, evaluated using (3.18) and (4.58) is also equal to ϵ. The negative energy contributes a value $-\epsilon/2$ so that the total energy is $\epsilon/2$. (This, of course, does not include the gravitational energy).

Coming back to the solution of the complex field equation (3.6), we may now state why we assumed $k = +1$ and did not consider the $k = 0$ and $k = -1$ cases. It can be seen that if we require the complex spacetime and the corresponding physical universe to have the same value of k, then the $k = 0,\ -1$ cases are unsuitable to describe the universe we livein. For $k = 0$, the constraint equation gives $\dot{x} = 0$, $\dot{y} = 0$, which leads to a static universe. For $k = -1$, it is true that we get a solution $\hat{a} = \pm t + ia_0$ from which $\mid \hat{a} \mid^2 = a_0^2 + t^2$ is obtainable. But in this case, the physical universe has to obey the equation

$$\frac{\dot{a}^2}{a^2} - \frac{1}{a^2} = -\frac{a_0^2}{a^4}; \tag{5.4}$$

i.e., the physical universe contain only the negative energy density. For these reasons, we do not consider these two possibilities as viable and set $k = +1$ at the outset.

5.2 Coasting Evolution

The physical models we obtain in both approaches (Secs. 3.2, 3.5) have coasting evolution. In the latter model, it is coasting throughout the history and in the former, it coasts when the universe is a few Planck times or more old. Historically, the first coasting cosmological model is the Milne universe. To understand this model, first consider a two dimensional flat spacetime given in coordinates (t, X) with the metric $ds^2 = dt^2 - dX^2$. Let the worldline L_0 be the line $X = 0$. By repeatedly using the Lorentz boost corresponding to some small velocity ΔV_0, a family of worldlines which all pass through O can be generated. A model in which these are the worldlines of fundamental observers represents an expanding universe obeying the cosmological principle. All the fundamental observers are equivalent to each other and because the worldline L_0 is a straight line representing inertial motion, the same is true for other worldlines too. Since the Lorentz boost is repeated infinitely often, an infinite number of worldlines are obtained by this construction. A four dimensional analogue of this model is usually referred to as Milne universe. This is a flat space cosmological model, not incorporating the effects of gravitation [66, 67]. Alternatively, it is described by a flat, empty spacetime having a RW metric with $k = -1$, $a(t) = t$ and $q = 0$.

Coasting cosmologies are encountered in many situations including non-Einstein theories of gravitation ([67] and references therein. In the Friedmann models itself, it is easy to see from (1.38) that coasting evolution results when $\rho + 3p = 0$, our models being examples. The quantity $\rho + 3p$ is sometimes referred to as gravitational charge. An interesting property of such spacetimes was recently pointed out by Dadhich *et al.* [68]. They resolve the Riemann tensor, which characterises the gravitational field into electric and magnetic parts, in analogy with the resolution of the electromagnetic field. It can be seen that the electric part is caused by mass-energy while the magnetic part is due to motion of the source. But unlike other fields, gravitation has two kinds of charges; one is the usual mass-energy and the other is the gravitational field energy. Consequently, also the electric part is decomposed into an active part, which is Coulombic and a passive part, which produces space curvature. An interchange of active and passive electric parts in the Einstein equation, which is referred to as electrogravity duality transformation, is shown to be equivalent to the interchange of Ricci and Einstein curvatures. These authors show that under this transformation, spacetimes with $\rho + 3p = 0$ go over to flat spacetime; i.e., they are dual to each other.

Absence of a particle horizon, agreement with the predicted age of the universe etc. in a coasting evolution are well known, but since it is usually considered as a feature of spacetimes containing only some exotic matter like strings, textures etc., this most simple cosmological scenario is not given serious attention in the literature. The Ozer-Taha model is coasting, but only upto the relativistic era and deviates from it after that epoch. Our physical model demonstrates that a coasting evolution throughout the history of the universe is a promising contender to a realistic cosmological model, which resolves all outstanding problems in the standard cosmology and at the same time not

making too drastic modifications to it.

5.3 Avoidance of Singularity

The physical model obtained in Sec. 3.3 is a bounce solution from a previous collapse, rather than an explosion from a big bang singularity. Such a bounce is sometimes referred to as a 'Tolman wormhole' [69, 70]. Oscillating universes have somewhat similar features and were considered as alternatives to the big bang cosmologies in the earlier literature, but interest in such cyclical evolution declined after the first cosmological singularity theorems. Recently, the quasi-steady state theory [71] revives this scenario. An analysis of bounce solutions reveals that the absolute minimum requirement for this to occur is the violation of (only) the strong energy condition (SEC). The various energy conditions, in the context of Friedmann models are the following [69]:

$$\begin{aligned}
\text{Null energy condition (NEC)} \quad &\Leftrightarrow \quad \rho + p \geq 0 \\
\text{Weak energy condition (WEC)} \quad &\Leftrightarrow \quad \rho \geq 0 \ \text{ and } \ \rho + p \geq 0 \\
\text{Strong energy condition (SEC)} \quad &\Leftrightarrow \quad \rho + 3p \geq 0 \ \text{ and } \ \rho + p \geq 0 \\
\text{Dominant energy condition (DEC)} \quad &\Leftrightarrow \quad \rho \geq 0 \ \text{ and } \ \rho \pm p \geq 0
\end{aligned} \tag{5.5}$$

It is shown that in a $k = +1$ universe, only the SEC need to be violated for obtaining a bounce solution. Since the singularity theorems mentioned above use the SEC as an input hypothesis, violating this condition vitiates them [73]. Physically, violating the other energy conditions with (small) quantum effects is relatively difficult. On the other hand, it is rather easy to violate the SEC and is therefore often referred to as 'the unphysical energy condition'. Using $\tilde{\rho}$ and $\tilde{p}$ given by (3.18) and (3.19) in the above energy conditions, we can see that our nonsingular model satisfies all the energy conditions except the strong one and serves as a perfect example for this phenomenon.

When comparing our two physical models, it is clear that the avoidance of singularity is primarily due to the presence of the negative energy density. The naturalness of a negative energy density at the classical level may be suspect. But we should note that the nonzero value of a_0 on which this depends is obtained on quantisation. However, as mentioned before, negative energy densities were postulated much earlier. Currently, there is a revival of interest in negative energies in connection with speculations on wormholes, time-travel etc. [72, 73]. Also some speculations are on which consider a Casimir driven evolution of the universe [74]. That negative energy densities are predicted by relativistic quantum field theory is known for a long time. Casimir [75], for the first time, showed that between two parallel perfect plane conductors separated by a distance l, there is a renormalised energy $E = -\pi^2/720l^3$ per unit area and this is now experimentally confirmed. The energy density corresponding to this may be evaluated as $-\pi^2/720l^4$. The Casimir energy density is calculated for some static

universe models. For example, this density for a massless scalar field in the four-dimensional static Einstein universe is [76]

$$\rho_{Casimir} = -\frac{0.411505}{4\pi^2 a^4}$$

A similar expression for an expanding closed universe is not known to us. However, we shall compare the above value with our expression for negative energy density (3.38), with a_0 given by (4.58): i.e.,

$$\rho_- = -\frac{1}{4\pi^2 a^4}.$$

Anyhow, it will be premature to identify ρ_- with Casimir energy, just like identifying ρ_λ with vacuum energy.

5.4 Prospects and Challenges

In this subsection, we discuss some of the possible future developments in connection with the new model, both observational and conceptual.

1) Consider the physical nonsingular model with real scale factor $a(t) = (a_0^2 + t^2)^{1/2}$. This model is obtainable directly from the assumption that the universe is closed and has a total energy density and pressure given by Eqs. (3.18) and (3.19) respectively. The assumption of complex scale factor etc. serves the purpose of justifying this one. It is shown that globally, the model has very good predictions and is devoid of all the cosmological problems mentioned in Sec. 2.1. But to be compatible with modern observational cosmology, it has to go a long way. Of utmost importance is the fluctuations in CMBR detected by COBE; any realistic cosmological model should be able to account for this. In Sec. 3.4, it was argued that the present model can generate density perturbations on scales as large as the present Hubble radius, even after the nucleosynthesis epoch. Recently, Coble *et al.* [45] have claimed that while models with a constant' cosmological constant have too high a COBE normalised amplitude for a scale invariant spectrum, their decaying-λ model has this amplitude matching with observations. However, a detailed analysis of CMBR anisotropies is not undertaken here. Another issue of importance which we have not looked into in any detail is the nucleosynthesis in the present model. It is shown that the thermal histories of the new model and the standard model are not very different. Hence it would be reasonable to expect that nucleosynthesis will also proceed identically.

2) If the standard model is to be generalised by including some kind of energy density other than relativistic/nonrelativistic matter, the resulting model cannot remain unambitious for long; it invariably has to get connected to field theory or the 'standard model' in particle physics. In that sense, the present model has only put forward a phenomenological law for the evolution of the vacuum energy which we prudently call λ (or ρ_λ). A field theoretic explanation for ρ_λ will always be welcome. In fact, one can see some resemblance between the set of equations (3.10)-(3.13) and (1.53)-(1.55),

which suggests the possibility of considering β as a field. It is easy to see that this is not an ordinary scalar field; it is more akin to a Brans-Dicke field. This aspect too is not pursued any further.

3) Another important issue worthy of further exploration is the connection with quantum stationary geometries' (QSG's) [77, 78]. As an example, this theory juxtaposes two situations; one in which a classical system of closed dust filled universe with constraint equation

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{A}{a^3} \tag{5.6}$$

having a singular evolution for the scale factor $a = a_{class.}(t)$ and the other in which QSG's avoid this singularity in such a way that

$$< a^2(t) >= a_0^2 + a_{class.}^2(t) \tag{5.7}$$

Also here, a_0 is shown to be of the order of Planck length. This is analogous to the avoidance of singularity in the new and its alternative models. This and many other aspects of the quantum behaviour in the model are left untouched.

Lastly, some aspects of aesthetics. It is well known that Einstein considered the right hand side of his equation, which contain a nongeometric quantity (the energy-momentum tensor) as spoiling the consistency and integrity of his geometrical approach. In the present case, we do not hesitate to claim that at least in a cosmological context, a realistic model is obtained in which such a voluntary introduction of a nongeometrical quantity is not necessary. In fact, equations (3.10)-(3.13) are essentially the same equations (3.5) and (3.6) and hence it can be considered that the right hand sides of (3.10)-(3.11) or that of (3.16)-(3.17) as emerging from their corresponding left hand sides.

One cannot simply be averse to the philosophical overtones of this theory. The universe with complex scale factor is the unperceived one, but the same field equations describe a real, physical universe with real scale factor. Our intellect can conceive only the measurable, real quantities and in a sense, this makes the energy-momentum tensor nonzero. If not approached with caution, this can lead to mysticism, but perhaps it would be better to interpret this, in the event of being proved to have some truth content, as yet another instance in physics where, to use N. Bohr's words, "truths being statements in which the opposite also are truths".

This position can be criticised on two grounds. (1) The observational and theoretical uncertainties are greatly amplified in cosmology and hence it is subjected to all sorts of ideological and philosophical influences, the present theory being one example. But it shall be reminded that none of the existing cosmological models are free from it and at the level of analysis made, the present model has equally good, if not better, predictions. (2) At a subtler level, it can be argued that it is our intellect that imposes its laws upon nature. We quote K. Popper [79], who remarked on this subject in reply to Kant: "Kant was right that it is our intellect which imposes its laws - its ideas, its values - upon the

inarticulate mass of our "sensations" and thereby brings order into them. Where he was wrong is that he did not see that we rarely succeed with our imposition, that we try again and again, and that the result - our knowledge of the world - owes as much to the resisting reality as to our self produced ideas". We note that this makes the task of conforming to any epistemological systematics difficult for the scientist.

As a closing note, we remark that the model with complex scale factor can be considered as a model for an underlying objective reality. The theory is clearly falsifiable; in the predictions $H_p t_p = 1$, $q_p = 0$, $\rho_m/\rho_\lambda = 2$ in the nonrelativistic era, the total energy density $\tilde{\rho}$, the negative energy density ρ_- etc., it leaves no adjustable parameters. Though it looks a mathematical curiosity, at best a toy model, it is curious enough how this simple model can account for this much cosmological observations without creating any problems at a physical level.

Bibliography

[1] Landau L D and Lifshitz E M 1975 *Classical Theory of Fields* (Oxford: Pergamon Press)

[2] Peebles P J E 1993 *Principles of Physical Cosmology* (Princeton: Princeton University Press)

[3] Weinberg S 1972 *Gravitation and Cosmology* (New York: Wiley)

[4] Kolb E W and Turner M S 1990 *The Early Universe* (Redwood City: Addison-Wesley)

[5] Narlikar J V 1993 *Introduction to Cosmology* (London: Cambridge University Press)

[6] Padmanabhan T 1996 *Cosmology and Astrophysics Through Problems* (Cambridge: Cambridge University Press)

[7] Adler S L 1982 *Mod. Phys.* **54** 729

[8] Narlikar J V and Padmanabhan T 1985 Creation field cosmology: A possible solution to singularity, horizon and flatness problems *Phys. Rev.* **D 32** 1928

[9] Dicke R H and Peebles P J E 1979 in: General Relativity: An Einstein centenery survey ed. Hawking S W and Israel W (London: Cambridge University Press)

[10] Rindler W 1956 *Mon. Not. R. Astr. Soc.* **116** 663

[11] Linde A 1990 *Particle Physics and Inflationary cosmology* (Switzerland: Harwood Academic Publishers)

[12] Brandenberger R H 1985 *Rev. Mod. Phys.* **57**, 1 and references therein

[13] Perlmutter *et al.* 1998 Measurements of Ω and Λ from 42 high-redshift supernovae *To appear in Ap. J., astro-ph /9812133*

[14] Kochanek C S 1996 Is there a cosmological constant? *Ap. J.* **466** 638

[15] Weinberg S 1989 The cosmological constant problem *Rev. of Mod. Phys.* **61** 1

[16] Hu Y, Turner M S and Weinberg E J 1994 Dynamical solutions to the horizon and flatness problems *Phys. Rev.* **D 49** 3830

[17] Liddle A R 1995 Inflation as the unique causal mechanism for generating density perturbations on scales well above the Hubble radius *Phys. Rev.* **D 51** 5347

[18] Sahni V 1998 The inflationary universe - from theory to observations *To appear in Pramana - J. Phys.*

[19] Guth A H 1981 Inflationary universe: A possible solution to horizon and flatness problems *Phys. Rev.* **D 23** 347

[20] Linde A D 1982 A new inflationary scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems *Phys. Lett.* **B 108** 389 ,

[21] Albrecht A and Steinhardt P J 1982 Cosmology for grand unified theories with radiatively induced symmetry breaking *Phys. Rev. Lett.* **48** 1220

[22] Linde A D 1983 Chaotic inflation *Phys. Lett.* **B 129** 177

[23] Freedman W L *et al.* 1994 Distance to the Virgo cluster galaxiy M100 from Hubble Space Telescopes observation of Cepheids *Nature* (London), **371** 757,

Pierce M J *et. al.* 1994 The Hubble constant and Virgo cluster distance from observations of Cepheid variables *Nature* (London), **371** 385 and also see [43]

[24] Lopez J and Nanopoulos D V 1996 A new cosmological constant model *Mod. Phys. Lett* **A 11** 1

[25] Ozer M and Taha M O 1986 A possible solution to the main cosmological problems *Phys. Lett.* **B 171** 363, 1987 A model of the universe free of cosmological problems *Nucl. Phys.* **B 287** 776

[26] Beesham A 1986 *Int. J. Theor. Phys.* **25** 1295

[27] Ratra B and Peebles P J E 1988 *Phys. Rev.* **D 37** 3407 Peebles P J E and Ratra B 1988 *Ap. J* **325** L17

[28] Gasperini M 1987 Decreasing vacuum temperature: A thermal approach to the cosmological constant problem *Phys. Lett.* **B 194** 347 , 1988 *Class. Quantum Grav.* **5** 521

[29] Freese K, Adams F C, Frieman J A and Mottola E 1987 Cosmology with decaying vacuum energy *Nucl. Phys.* **B 287** 797

[30] Olson T S and Jordan T F 1987 Ages of the universe for decreasing cosmological constants *Phys. Rev.* **D 35** 3258

[31] Reuter M and Wetterich C 1987 *Phys. Lett.* **B 188** 38

[32] Chen W and Wu Y 1990 Implications of a cosmological constant varying as R^{-2} *Phys. Rev.* **D 41** 695

[33] Berman M S 1991 Cosmological models with a variable cosmological term *Phys. Rev.* **D 43** 1075

[34] Pavon D 1991 Nonequilibrium fluctuations in cosmic vacuum decay *Phys. Rev.* **D 43** 375

[35] Carvalho J C Lima J A S and Waga I 1992 *Phys. Rev.* **D 46** 2404

[36] Abdel-Rahman A-M M 1992 Singularity free decaying vacuum cosmologies *Phys. Rev.* **D 45** 3497

[37] Salim J M and Waga I 1993 Thermodynamic constraints on time-dependent Λ model *Class. Quantum Grav.* **10** 1767

[38] Waga I 1993 Decaying vacuum flat cosmological models: Expressions for some observable quantities and their properties *Ap. J.* **414** 436

[39] Calvao M O, Oliveira H P, Pavon D and Salim J M 1992 Dissipative cosmology with decaying vacuum energy *Phys. Rev.* **D 45** 3869

[40] Maia M D and Silva G S 1994 *Phys. Rev.* **D 50** 7233

[41] Lima J A S and Maia J M F 1994 Deflationary cosmology with decaying vacuum energy density *Phys. Rev* **D 49** 5597

[42] Matyjasek J 1995 Cosmological models with a time-dependent Λ term *Phys. Rev.* **D 51** 4154

[43] Lima J A S and Trodden M 1996 Decaying vacuum energy and deflationary cosmology in open and closed universes *Phys. Rev.* **D 53** 4280

[44] Abdussattar and Viswakarma R G 1997 Some FRW models with variable G and Λ *Class. Quantum Grav.* **14** 945

[45] Coble K, Dodelson S and Friemann J 1997 Dynamical Λ models of structure formation *Phys. Rev.* **D 55** 1851

[46] Newman E T 1988 The remarkable efficacy of complex methods in general relativity in: *Highlights in Gravitation and Cosmology* eds. Iyer B R, Kembhavi A, Narlikar J V and Vishveshwara C V (Cambridge: Cambridge University Press)

[47] Flaherty E J, Jr.1980 in: General Relativity and Gravitation- One hundred years after the birth of Albert Einstein, ed. Held A (New York:Plenum)

[48] Hartle J B and Hawking S W 1983 Wave function of the universe *Phys. Rev.* **D 28** 2960

[49] Ellis G, Sumeruk A, Coule D and Hellaby C 1992 Change of signature in classical relativity *Class. Quantum Grav.* **9** 1535

Ellis G F R 1992 Covariant signature change in classical relativity *Gen. Relativ. Gravit.* **24** 1047

[50] Hayward S A 1992 Signature change in general relativity *Class. Quantum Grav.* **9** 1851

[51] Greensite J 1993 *Phys. Lett.* **B 300** 34

Carlini A and Greensite J 1993 Why is space-time Lorentzian? *Phys. Rev.* **D 49** 866

[52] Hayward S A 1996 Complex lapse, complex action and path integrals *Phys. Rev.* **D 53** 5664

[53] V. John M and Babu Joseph K 1996 A modified Ozer-Taha type cosmological model, *Phys. Lett.* **B 387** 466

[54] V. John M and Babu Joseph K 1997 A low matter density decaying vacuum cosmology from complex metric *Class. Quantum Grav.* **14** 1115

[55] Hoyle F and Narlikar J V 1963 *Proc. R. Soc. London* **A273** 1, 1966 **A290** 143, 1966 **A 290** 166 and also see [8].

[56] Kolb E W 1989 A coasting cosmology *Ap. J* **344** 543

[57] Kamionkowski M and Toumbas N 1996 A low density closed universe *Phys. Rev. Letters* **77** 587 and references therein.

[58] Vilenkin A 1984 *Phys. Rev. Letters* **53** 1016

Gott III R J and Rees M J 1987 Astronomical constraints on a string dominated universe *Mon. Not. R. Astr. Soc.*227 453

[59] Lifshitz E M and Khalatnikov I M 1963 *Adv. Phys.* **12** 185

[60] Halliwell J J 1991 Introductory lectures on quantum cosmology *Proc. Jerusalem Winter School on Quantum Cosmology* ed. T. Piran *et. al* (Singapore: World Scientific) and references therein.

[61] Vilenkin A 1998 The wave function discord gr-qc/9804051

[62] Padmanabhan T 1989 Some fundamental aspects of semiclassical and quantum gravity *Int. J. Mod. Phys.* **A 4** 4735

[63] Linde A D 1984 *Lett. Nuovo Cimento* **39** 401

[64] Vilenkin A 1984 *Phys. Rev.* **D30** 509
Vilenkin A 1986 *Phys. Rev.* **D33** 3560

[65] Fil'chenkov M L 1995 *Phys. Lett.* **B 354** 208

[66] Ellis G F R and Williams R M 1988 *Flat and Curved Spacetime* (Oxford: Oxford University Press)

[67] Sahni V 1984 Regularised quantum energy-momentum tensor for spinor fields in the Chitre-Hartle and MIlne metrics *Class Quantum Grav.* **1** 579

[68] Dadhich N, Patel L K and Tikekar R 1998 A duality relation for fluid spacetime *To appear in Class. Quantum Grav.*

[69] Molina-Paris C and Visser M 1998 Minimal conditions for the creation of a Friedmann-Robertson-Walker universes from a "bounce" *gr-qc/9810023*

[70] Coule D H 1993 Wormhole or bounce? *Class. Quantum Grav.* **10** L25

[71] Narlikar J V 1996 *Elements of Cosmology* (Hyderabad: Universities Press (India)Ltd.) and references therein.

[72] Morris M S, Thorne K S and Yurtsever U 1988 Wormholes, time machines and the weak energy condition *Phys. Rev. Lett.* **13** 1446

[73] Visser M 1995 *Lorenzian Wormholes* (AIP Press)

[74] Antonsen F and Bormann K 1998 Casimir driven evolution of the universe *gr-qc/9802013*

[75] Casimir H B G 1948 On the attraction between two perfectly conducting plates *Proc. Kon. Ned. Akad. Wet.* **B 51** 793

[76] Elizalde E 1994 The vacuum energy density for spherical and cylinderical universes *J. Math. Phys.* **35** 3308

[77] Padmanabhan T and Narlikar J V 1981 Stationary states in a quantum gravity model *Phys. Lett.* **A 84** 361

[78] Padmanabhan T 1983 An approach to quantum gravity *Phys. Rev.* **D 28** 745, 1983 Universe before Planck time: A quantum gravity model *Phys. Rev.* **D 28** 756

[79] Popper K R 1973 *Objective Knowledege* (Oxford: Clarendon Press)